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### The target-incentive system vs. the price-incentive system under adverse selection and the ratchet effect

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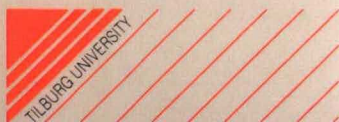
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by  
Liang Zou

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# The target-incentive system vs. the price-incentive system under adverse selection and the ratchet effect

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In a dynamic, adverse selection type of principal–agent relationship, where the principal is committed over time to a particular *incentive system* (with fixed structure but variable parameters) but not to any intertemporal *incentive scheme* (with both fixed structure and parameters), the ratchet effect arises but varies with different incentive systems. We show that, compared with using prices as the sole planning mode, the principal can attenuate the ratchet effect by stipulating targets as well. As a result, the target-incentive system entails higher expected welfare than the price-incentive system in a perfect Bayesian equilibrium.

## 1. Introduction

In large economic organizations the rights to make production decisions are often delegated to subordinates (agents) engaging in production. The central authority (the principal) performs the role of coordination and maintains control through incentive schemes that specify the payments to the agents based on their performance, taking into account that the agents may have diverse interests which are in conflict with that of the organization as a whole, and have private information which is (prohibitively) costly to obtain for the center. It is the very difference in access to information between the principal and the agents that justifies delegating production decisions [see Holmström (1982)].

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Real world incentive schemes most often take the form of a linear function: the central authority sets up a price for a product and either 'purchases' the whole output or lets the agent sell the product in the market at this price. There is usually a lump-sum transfer of funds involved, taking the form of a 'subsidy' or 'tax' according to its sign. We call such schemes 'price-incentive schemes', and the class of such schemes a 'price-incentive system' (PIS). The simplicity of the incentive schemes in practice can be partly explained by the costs of writing complex schemes, but the principal-agent theory also provides rich examples where price-incentive schemes are in fact optimal.<sup>1</sup> Another advantage of the price-incentive schemes is their robustness to forecasting errors.

However, these advantages notwithstanding, the PIS often restricts the central authorities' power to explore all the possibilities to maximize the organizational welfare.<sup>2</sup> This gives rise to the interest of looking at other incentive systems that preserve most or all of the good properties of PIS but allow the principal to non-trivially improve welfare.

In this paper we examine the welfare benefit to the principal of adding a minimal required target variable to a price-incentive scheme. We call the class of such schemes a 'target-incentive system' (TIS) and each element of TIS a 'target-incentive scheme'. More precisely, we allow the principal to set a target of output and reward the agent according to a price (piece-rate) with a lump-sum transfer only when the realized output is above or equal to the target. This model applies to situations where the principal is quite sure about the agent's capacity to achieve a certain production level. For example, assume that the output is relatively independent of other non-observable technological states and is only affected by the agent's hidden production capacity, and that in the preceding period the agent has produced an output level  $x$ , then the principal can require the agent to produce at least  $x$  in the coming period.

Precise definitions of the PIS and TIS are given in the next section. It is reasonable to assume that costs of writing a target-incentive scheme are more or less at the same level of writing a price-incentive scheme, since targets are usually easy to describe and understand. Since the minimal target can always be set at zero, the TIS is obviously a more general class of schemes than the

<sup>1</sup>In moral hazard models optimal incentive schemes are linear when the agent is risk neutral [e.g. Harris and Raviv (1979)]. In simultaneous moral hazard and adverse selection models the optimal incentive mechanisms are a family of linear incentive schemes [e.g. Laffont and Tirole (1986)]. Recently Holmström and Milgrom (1987) obtained further insight into the optimality of linear incentive schemes. They showed that, even with risk-averse agents, the optimal incentive schemes should be linear functions of the observable variables when the agent's private action space is sufficiently rich.

<sup>2</sup>Under asymmetrical information the second-best incentive schemes are rarely linear [e.g. see Guesnerie and Laffont (1984) for adverse selection problems, Maskin and Riley (1984) for monopoly pricing problems, Mirrlees (1971) for non-linear taxation problems].



PIS and contains the PIS. Therefore, if the principal-agent relationship lasts for only one period, the TIS is at least weakly superior to the PIS: any price-incentive scheme is just a particular target-incentive scheme with a zero target level. In fact this superiority is strict (Proposition 4).

However, most economic organizations are long-lasting in nature, where planning and control activities happen periodically. The dominance of TIS over PIS becomes doubtful in these repeated relationships. For one thing, enlarging the principal's control set might aggravate the so-called 'ratchet effect' and even the static superiority of the TIS might be upset by the agent's strategic behavior. In this context, anticipating the principal's tendency to update targets using revealed information, the agent may be induced to choose suboptimal output levels in earlier periods in order to disguise his true information and maximize the long-term information rents [e.g. Weitzman (1980)]. Of course, similar strategic behavior may occur in the PIS as well if the principal updates the prices using revealed information [e.g. Freixas et al. (1985)]. Therefore we have to be explicit about these strategic factors before drawing any conclusion concerning the relative long-term welfare implications of the TIS and PIS.

Formally we consider a two-period principal-agent relationship. The principal is committed over time to one of the two incentive systems, the PIS and the TIS, and chooses an incentive scheme in each period from the pre-committed incentive system.<sup>3</sup> We assume that the principal cannot commit to any intertemporal incentive schemes, thus ratchet effects will arise and vary with the committed systems. Our interest is to see how expected organizational welfare is affected by the alternative policies of committing to the PIS and the TIS. Recent studies of the ratchet effects provide useful tools for our analysis [e.g. Laffont and Tirole (1987, 1988), Freixas, Guesnerie and Tirole (1985)]. Our model is constructed to be merely a slightly generalized version of the one analyzed by Freixas et al. (1985); in this way, we are able to capitalize on some of their analytical results.

An interesting observation in this paper is that, although it is the widespread use of targets in centrally planned economies that has attracted attention to the ratchet effect, the use of targets can actually mitigate such effect if the alternative is to use only prices (with lump-sum transfers) as planning instruments. That is, if the principal is deprived of the right to set targets, and is constrained to choose incentive schemes in the PIS, the ratchet effect can be aggravated (in a probabilistic sense, Proposition 7). Put differently, we find here an instance that commitment to the more restrictive policies can hurt the principal and the organization as a whole, though the agents will generally benefit. At first glance this result appears to be at odds

<sup>3</sup>For instance, assume that an incentive system has been established and enforced by law before the principal assumes his authority.

with an insight in bargaining theory<sup>4</sup> that it can be advantageous for a bargaining party to 'tie its hands' by committing to a more restrictive strategic set or "'incentive" system' [in the sense of Schelling (1956, p. 300)]. The reason for this apparent 'incongruity' in fact lies on the other side of the coin. One just has to beware that tying one's hands does not always help: it can hurt as well. In the present context, if the principal commits himself to not using targets, i.e. to the PIS, the high productivity agent will enjoy higher information rents than his rents in the TIS. Thus, if the agent reveals his information in the first period, he will suffer a greater loss than in the TIS in the second period. And this gives the agent a stronger incentive to 'cheat' in the first period in the PIS, which leads to a greater welfare loss for the organization. Therefore if the TIS is available, it is never judicious for the principal to commit to the more restrictive PIS.

The model is presented in section 2. As a preliminary step, complete information results are derived. It is shown that with complete information and certainty the TIS and the PIS are essentially identical since they both entail the same level of welfare. Section 3 is devoted to derivations and analyses of the optimal incentive schemes under single-period planning with adverse selection. The strict welfare dominance of TIS over PIS in the static relationship is proved in Proposition 4. Properties characterizing the optima are also examined. In section 4 we first formulate the dynamic incentive problem, then show in Proposition 6 the existence of a perfect Bayesian equilibrium in both incentive systems. We then argue that for any first-period incentive system, more desirable equilibria can result if there is precommitment to the use of TIS, rather than PIS, in the second period (Proposition 7). The dynamic welfare dominance of the TIS over the PIS is eventually proved in Proposition 9 and Proposition 10. Section 5 contains some concluding remarks and conjectures.

## 2. The model

Consider a basic organizational structure with a principal and an agent interacting in a decentralized planning environment. The relationship may be best thought of as that between a central planner and a public firm manager, although it is also suitable to be conceived as between a regulatory department and a controlled private firm, between headquarters and a subsidiary, etc. There are two periods  $t=1,2$ , technologically identical.<sup>5</sup> Let  $y_t \in Y = R_+$  be a performance indicator for the agent in period  $t$ , publicly

<sup>4</sup>See, for example, the classic essay of Schelling (1956).

<sup>5</sup>By this assumption we exclude the possibility of having potential gains from renegotiation in the presence of technological change.



observable.  $y_t$  might be taken either as the output of a particular good, or simply the publicly observable revenue of the agent. Producing  $y_t$  costs the agent  $C(y, \theta)$  ( $C_y > 0$ ,  $C_{yy} > 0$ , and yields a payoff for the organization,  $N(y_t)$  ( $N' > 0$ ,  $N'' \leq 0$ ), both measured in terms of money. The coefficient  $\theta$  in the cost function stands for technology, and may take two values:  $\theta = L$  or  $\theta = H$  ( $L < H$ ). Assume  $C(y, L) > C(y, H)$  and  $C_y(y, L) > C_y(y, H)$  for all  $y \in Y$ , i.e. with technology  $L$  both total and marginal costs are higher. Thus,  $H$  and  $L$  denote 'high productivity' and 'low productivity', respectively.

The agent knows exactly the value of  $\theta$  from the outset; the principal's knowledge of  $\theta$  is limited, and is characterized by a prior belief  $v_1 = \text{Prob}\{\theta = L\}$ . This belief is assumed to be common knowledge. The actual cost incurred at the end of each period is not observed by the principal; were this to be the case the problem would be trivial. In each period  $t$ , the principal offers the agent an incentive scheme  $S_t(\cdot): Y \rightarrow R$ , which specifies the payment to the agent as a function of  $y_t \in Y$ . The agent may accept or reject the offer in each period. In the case of a rejection, there is no further production.

The *price-incentive system* (PIS), denoted  $\Gamma^P$ , is defined as the set of incentive schemes

$$\Gamma^P = \{S(\cdot) \mid S: y \in Y \mapsto S(y) = a + by \in R; a, b \in R\}, \quad (1)$$

where  $a$  may be interpreted as a lump-sum tax or subsidy according to its sign, and  $b$  a controlled or transfer price for the good produced. The superscript  $P$  denotes the price system. We see that  $\Gamma^P$  is merely a class of linear functions defined on  $Y$ .

The *target-incentive system* (TIS), denoted  $\Gamma^T$ , is defined as the set of incentive schemes

$$\Gamma^T = \left\{ S(\cdot) \mid S: y \in Y \rightarrow S(y) = \begin{cases} \alpha + \beta(y - \tau) \in R, & \text{if } y \geq \tau, \\ 0 \in R, & \text{if } y < \tau, \end{cases} \quad \tau > 0, \alpha, \beta \in R \right\}, \quad (2)$$

where  $\tau$  denotes the target,  $\alpha$  the lump-sum transfer of funds, and  $\beta$  the piece-reward for over-fulfillment of the target. The penalty for under-production is formalized as the agent being deprived of any compensation. It can be perceived as an approximation of the situations in which there is a rigid quota, floor or ceiling, etc. and violation in the wrong direction is not allowed. Since the agent is assumed to be perfectly informed, under-production (below the target level) will never happen and thus the actual level of penalty is not essential. The elements of  $\Gamma^P$  and  $\Gamma^T$  will be denoted by  $S^P$  and  $S^T$ , and called price- and target-incentive schemes, respectively.

For notational convenience we introduce a variable  $h \in \{P, T\}$ ; when  $h$  is

used as a superscript, the related expression is meant to be suitable for both incentive systems. Assume that the agent is a profit maximizer with a zero level of reservation profit (without loss of generality). The  $t$ -period profit for agent  $\theta$  given an incentive scheme  $S_t^h \in \Gamma^h$  is

$$\pi_t^h(S_t^h, y_t^h, \theta) = S_t^h(y_t^h) - C(y_t^h, \theta). \quad (3)$$

The principal is concerned with maximizing (social) welfare, which is assumed to be the gross payoff,  $N(y_t^h)$ , less the cost of production,  $C(y_t^h, \theta)$ , and less a measure of the undesirable effect caused by the money transfer. A standard formalization of this effect, which we shall employ, is to associate a unit cost  $\lambda > 0$  to the funds transferred to the agent.<sup>6</sup> Formally, the  $t$ -period welfare given  $S_t^h \in \Gamma^h$  for each  $\theta$  is<sup>7</sup>

$$W_t^h(S_t^h, y_t^h, \theta) = N(y_t^h) - C(y_t^h, \theta) - \lambda S_t^h(y_t^h). \quad (4)$$

Let  $\delta$  be the common discount factor. With straightforward notation, the total discounted profit and the total discounted welfare are:

$$\pi^h(S_1^h, S_2^h, y_1^h, y_2^h, \theta) = \pi_1^h(S_1^h, y_1^h, \theta) + \delta \pi_2^h(S_2^h, y_2^h, \theta) \quad (5)$$

and

$$W^h(S_1^h, S_2^h, y_1^h, y_2^h, \theta) = W_1^h(S_1^h, y_1^h, \theta) + \delta W_2^h(S_2^h, y_2^h, \theta). \quad (6)$$

### 2.1. Optimal results under complete information

It is useful to first examine the case where the value of  $\theta$  is common knowledge. Assume that the production is profitable for both types of agents. In  $\Gamma^h$ , the principal's objective is to design a sequence of schemes  $S_1^h$  and  $S_2^h$  so that the total discounted welfare given by (6) is maximized subject to  $\pi_t^h \geq 0$ ,  $t = 1, 2$ . Let  $y_t^h(S_t^h, \theta)$  be the  $\theta$  agent's  $t$ -period profit-maximizing output. Clearly, given complete information, the principal only needs to solve for  $S_t^h$  in each period  $t$ ,  $t = 1, 2$ , from the program

<sup>6</sup>There are many reasons why transferring funds is costly: raising funds through taxation may have distortionary effects; allowing the agent to reap extra profit (net of its production cost) may have 'political costs' for an egalitarian government, etc. Note also that our (social) welfare function is analytically equivalent to the welfare function formalized as a weighted sum of the consumers' benefit plus the profit for the firm (agent) as in Baron and Myerson (1982). See Freixas et al. (1985) for more discussions.

<sup>7</sup>For conciseness we use the same notation,  $S_t^h$ , to denote both an incentive scheme and a particular payment. Similarly,  $y_t^h$  may denote both agents' strategic response to an incentive scheme and a particular output level. The context will help exclude any confusion.

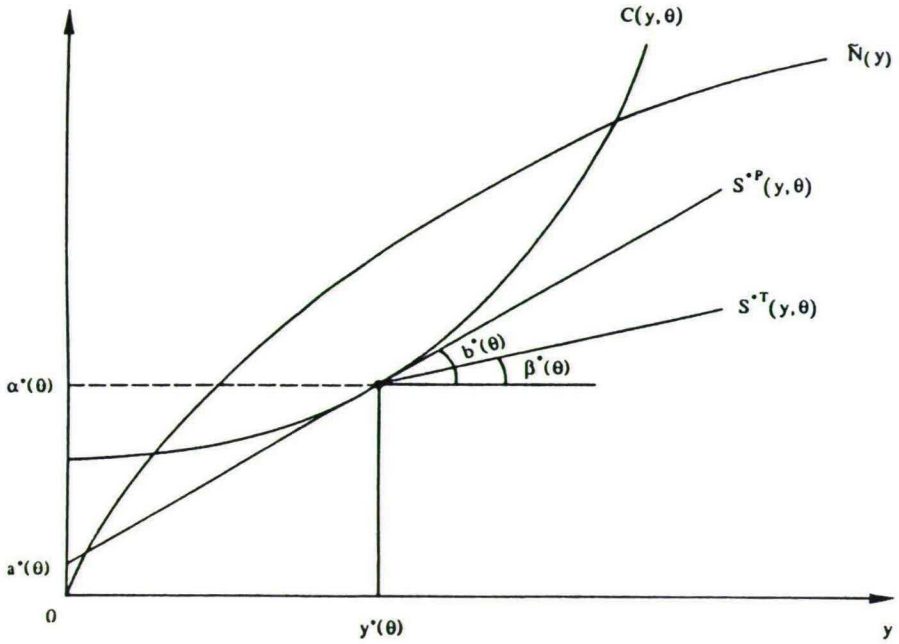


Fig. 1

$$\max_{S_t^h \in F^h} W_t^h(S_t^h, y_t^h(S_t^h, \theta), \theta) \quad (7)$$

$$\text{s.t. } \pi_t^h(S_t^h, y_t^h(S_t^h, \theta), \theta) \geq 0. \quad (8)$$

One can easily verify (see fig. 1) that there is a unique optimal output  $y^*(\theta)$ , identical in both periods, that maximizes  $W_t^h$  subject to (8), and satisfies

$$C_y(y^*(\theta), \theta) = N'(y^*(\theta))/(1 + \lambda) \triangleq \tilde{N}'(y^*(\theta)). \quad (9)$$

The optimal output,  $y^*(\theta)$ , can be induced through the optimal incentive schemes in either incentive system. We call  $\tilde{N}(y)$  the *virtual payoff function*. Fig. 1 shows that at  $y^*(\theta)$ , the vertical distance between  $\tilde{N}(y)$  and  $C(y, \theta)$ , is maximized, and (9) says that this optimal output is characterized by the equality of marginal cost and the marginal virtual payoff of production. The optimal incentive schemes are given by (see fig. 1)

$$S_t^{*P}(y_t, \theta) = S^{*P}(y_t, \theta) = a^*(\theta) + b^*(\theta)y_t,$$



where

$$b^*(\theta) = C_y(y^*(\theta), \theta), \quad (10)$$

$$a^*(\theta) = C(y^*(\theta), \theta) - b^*(\theta)y^*(\theta), \quad (11)$$

and

$$S_t^{*T}(y, \theta) = S^{*T}(y, \theta) = \begin{cases} \alpha^*(\theta) + \beta^*(\theta)(y_t - \tau^*(\theta)), & \text{if } y_t \geq \tau^*(\theta), \\ 0, & \text{if } y_t < \tau^*(\theta), \end{cases}$$

where

$$\tau^*(\theta) = y^*(\theta), \quad (12)$$

$$\alpha^*(\theta) = C(y^*(\theta), \theta), \quad (13)$$

$$\beta^*(\theta) \leq C_y(y^*(\theta), \theta). \quad (14)$$

If  $b^*(\theta)$  satisfies (10), the agent chooses  $y^*(\theta)$  as his profit-maximizing output. Eqs. (11) and (13) are equivalent to  $\pi_t^h(S^{*h}, y^*(\theta), \theta) = 0$ ,  $t = 1, 2$ , i.e. the agent is put at the minimal profit level because transferring funds is costly. Eq. (12) says that the target equals the desired output, and (14) ensures that the agent has no incentive to deviate from the targeted output. Henceforth we denote the single-period optimal welfare under complete information by  $W^*(\theta) = W^h(S^{*h}(y^*(\theta), \theta), y^*(\theta), \theta)$ . The total discounted welfare is consequently  $(1 + \delta)W^*(\theta)$ . It is clear that in an environment with complete information the PIS and the TIS entail the same level of welfare.

### 3. Incentive systems in a one-period relationship with incomplete information

In this section we study the case where the principal-agent relationship lasts for only one period. We use the notation introduced in the preceding section, except that the subscript  $t$  is omitted.

Under incomplete information it may be optimal to shut down agent  $L$  when  $v$  is small. Let  $\underline{v}^P$  and  $\underline{v}^T$  denote the 'cut-off' belief in the PIS and TIS, respectively: if  $v \leq \underline{v}^T$  (or  $\underline{v}^P$ ) agent  $L$  will be induced to quit; otherwise both types of agent will produce. Our research strategy is as follows. We first analyze the problem as though a cut-off belief,  $\underline{v}^h$ , exists and is given in each incentive system, and focus on the cases where  $v > \underline{v}^h$ , i.e. where both agents' individual rationality constraints must be satisfied. Then, when sufficient insights are obtained, we show the existence of such cut-off beliefs. Let  $\underline{v} = \max \{\underline{v}^T, \underline{v}^P\}$ .

Now let a cut-off belief,  $\underline{v}^h$ , be given for  $h = P, T$ . Suppose  $v \in (\underline{v}^h, 1)$ . Given

an incentive scheme  $S^h \in \Gamma^h$ , and the agent's optimal response strategy,  $y^h(S^h, \theta) \in Y$ , the expected single-period welfare is

$$E(W^h | S^h, v) \stackrel{\text{def}}{=} v W^h(S^h, y^h(S^h, L), L) + (1 - v) W^h(S^h, y^h(S^h, H), H). \quad (15)$$

The principal's problem is given by

$$\max_{S^h \in \Gamma^h} E(W^h | S^h, v) \quad (16)$$

$P^h$ :

$$\text{s.t. } \pi^h(S^h, y^h(S^h, \theta), \theta) \geq 0, \quad \theta = L, H. \quad (17)$$

The following assumptions on the cost structure are made to ensure that the second-order conditions for program  $P^h$  are met.

*Assumption 1.*  $C_{yy}(y, \theta) \geq 0$ , for all  $y \in Y$ ,  $\theta = L, H$

*Assumption 2.* For  $x \in Y$  and  $z \in Y$  such that  $C_y(x, L) = C_y(z, H)$ , the cost functions satisfy  $(1 + 2\lambda)C_{yy}(x, L) > \lambda C_{yy}(z, H)$ .

Assumption 1, (29), and (20) imply  $y'' \leq 0$ , i.e. the marginal rate of output w.r.t. reward or price is non-increasing. Since usually  $\lambda$  is small, Assumption 2 is not as restrictive as it appears. Assumptions 1 and 2 are maintained throughout this paper.

### 3.1. Optimal solutions to $P^T$ in the TIS

Clearly, given  $S^T$ , no under-fulfillment of the target may occur. The profit-maximizing output,  $y^T(S^T, \theta)$ , for agent  $\theta$ , given that the resulting profit is non-negative, must satisfy

$$y^T(S^T, \theta) = \tau, \quad \text{iff } C_y(\tau, \theta) \geq \beta, \quad (18)$$

$$y^T(S^T, \theta) = y(\beta, \theta), \quad \text{iff } C_y(\tau, \theta) < \beta, \quad (19)$$

where  $y(\beta, \theta)$  is defined by

$$C_y(y(\beta, \theta), \theta) = \beta. \quad (20)$$

We use the triple  $(\tau, \alpha, \beta)$  interchangeably with  $S^T$  to designate a target-incentive scheme. To facilitate derivation we first single out a simple fact concerning an optimal  $(\tau, \alpha, \beta)$ .

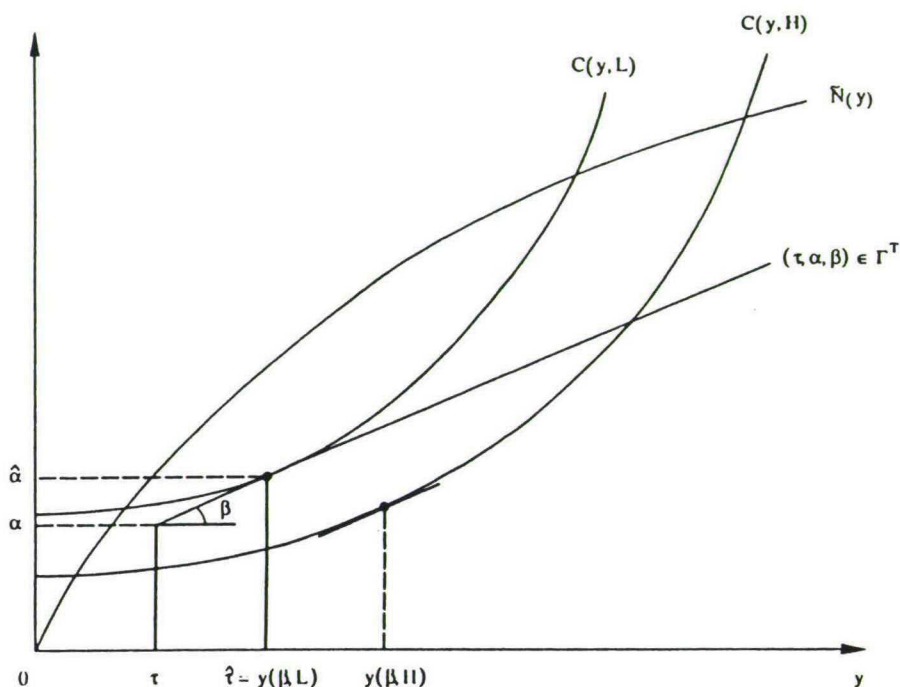


Fig. 2

**Lemma 1.** *In search of a solution  $(\tau, \alpha, \beta)$  to program  $P^T$ , there is no loss of generality to restrict attention to the schemes satisfying  $C_y(\tau, H) \leq \beta \leq C_y(\tau, L)$ .*

*Proof.* For any target scheme  $(\tau, \alpha, \beta)$  with  $\beta > C_y(\tau, L)$  (see fig. 2), agent  $L$  will choose to produce  $y(\beta, L)$ , which satisfies condition (20). We may design another scheme,  $(\hat{\tau}, \hat{\alpha}, \hat{\beta})$ , with  $\hat{\tau} = y(\beta, L)$ ,  $\hat{\alpha} = \alpha + \beta(y(\beta, L) - \tau)$ , and  $\hat{\beta} = \beta$ . It is easily seen that  $(\hat{\tau}, \hat{\alpha}, \hat{\beta})$  achieves the same result as does  $(\tau, \alpha, \beta)$  but with  $\hat{\beta} \leq C_y(\hat{\tau}, L)$ .

For a target scheme with  $\beta < C_y(\tau, H)$ , since both types of agent will react by producing exactly the assigned target  $\tau$ , it does not alter the choice of either agent, nor their payments, if  $\beta$  increases to  $\hat{\beta} = C_y(\tau, H)$ . Q.E.D.

Lemma 1 implies that the principal can rely on target-incentive schemes that induce agent  $L$  to just fulfill the imposed target. In fact this is necessary for a target-incentive scheme to be an optimal solution to  $P^T$  (Proposition 2).

The lump-sum payment,  $\alpha$ , should be set so as to make at least one of the constraints in (17) binding. The binding constraint must be that of agent  $L$ , since agent  $H$  can always produce  $y^T(S^T, L)$  with a lower cost. Thus

$$\alpha = C(\tau, L). \quad (21)$$

By Lemma 1 and the conditions (18)–(21), we may write  $y^T(S^T, L) = \tau$ ,  $y^T(S^T, H) = y(\beta, H)$ , and reduce  $P^T$  to

$$\begin{aligned} \max_{\tau, \beta} E(W^T | (\tau, \beta), v) \stackrel{\text{def}}{=} & v[N(\tau) - (1 + \lambda)C(\tau, L)] \\ & + (1 - v)[N(y(\beta, H)) - C(y(\beta, H), H) \\ & - \lambda(C(\tau, L) + \beta(y(\beta, H) - \tau))] \end{aligned} \quad (22)$$

$$\text{s.t. } C_y(\tau, H) \leq \beta \leq C_y(\tau, L). \quad (23)$$

*Proposition 1.* For  $v \in (v^T, 1)$ , there exists a unique target scheme  $(\tau(v), \alpha(v), \beta(v))$  which solves the program (22)–(23); it is differentiable in  $v$  on  $(v^T, 1)$  and satisfies (21) and

$$C_y(\tau, L) = \frac{1}{\lambda + v} (v(1 + \lambda)\tilde{N}'(\tau) + (1 - v)\lambda\beta), \quad (24)$$

$$(\tilde{N}'(y(\beta, H)) - \beta)y'(\beta, H) = \frac{\lambda}{1 + \lambda} (y(\beta, H) - \tau). \quad (25)$$

*Proof.* See the appendix. The proof uses Assumptions 1 and 2 and amounts to a straight-forward check of the first- and second-order conditions. From the characterization in Proposition 1 we derive some useful properties regarding the optimal results:

*Proposition 2.* If  $(\tau, \alpha, \beta)$  solves  $P^T$ , then

$$(i) \quad \tilde{N}'(y(\beta, H)) > C_y(y(\beta, H), H) = \beta, \quad (26)$$

$$(ii) \quad \tilde{N}'(\tau) > C_y(\tau, L) > \beta. \quad (27)$$

*Proof.* (i) (18)–(20) and (21) imply  $y(\beta, H) \geq \tau$ . If  $y(\beta, H) = \tau$ , then from (20)  $C_y(\tau, H) = \beta$ . This, however, cannot be true, for else by (24) and (25) we would have  $C_y(\tau, L) = \beta = C_y(\tau, H)$ , which contradicts the assumption  $C_y(y, L) > C_y(y, H)$  for all  $y \in Y$ . Thus,  $y(\beta, H) > \tau$ . It follows from (20) that  $y'(\beta, H) > 0$  and from (25) that

$$\tilde{N}'(y(\beta, H)) - C_y(y(\beta, H), H) = \tilde{N}'(y(\beta, H)) - \beta$$



$$= \frac{\lambda}{1+\lambda} \frac{y(\beta, H) - \tau}{y'(\beta, H)} > 0.$$

(ii) From  $\tilde{N}'' \leq 0$  and (i),  $\tilde{N}'(\tau) > \beta$ . This fact with (24), which says that  $C_y(\tau, L)$  is a convex combination of  $\tilde{N}'(\tau)$  and  $\beta$ , immediately implies (27). Q.E.D.

It is easy to derive from Proposition 2 that for a solution  $(\tau, \alpha, \beta)$  to  $P^T$ ,  $C_y(\tau, H) < \beta < C_y(\tau, L)$ ; i.e. constraint (23) is not binding. We saw in the previous section that under complete information either (26) or (27) must hold with equality for an optimal incentive scheme, depending on which  $\theta$  occurs. Under incomplete information, however, both relations cannot be satisfied by a single scheme; hence, there has to be a compromise between the two. This is given simultaneously in (24) and (25). In (24) the marginal cost for agent  $L$ ,  $C_y(\tau, L)$ , is a weighted average of the marginal virtual payoff,  $\tilde{N}'(\tau)$ , and the piece-rate of reward,  $\beta$ . The weight assigned to  $\tilde{N}'(\tau)$  and  $\beta$  varies with the principal's belief  $v$ , the probability that  $\theta = L$ .  $C_y(\tau, L)$  is closer to  $\tilde{N}'(\tau)$  as  $v$  becomes larger and equals  $\tilde{N}'(\tau)$  when  $v$  takes the extreme value of one. In (24) and (25), if the unit cost of funds  $\lambda$  goes to zero, the equations become identical to the characterization of the complete information output, (10) and (14), which implies  $\tau = y^*(L)$  and  $y(\beta, H) = y^*(H)$ . This corresponds to a Groves mechanism<sup>8</sup> which achieves socially efficient production in both states  $L$  and  $H$  but allows agent  $H$  to reap (probably enormous) rents. Proposition 2 implies also that the marginal cost of production is strictly smaller than the marginal virtual payoff for both  $\theta = L$  and  $\theta = H$ , which implies  $\tau < y^*(L)$  and  $y(\beta, H) < y^*(H)$ . The relative magnitudes of the various output levels are shown in fig. 3. To obtain some intuition, it is useful to rewrite the principal's utility (for a given  $\theta$ ) as

$$\begin{aligned} W^h(S^h, y^h, \theta) &= N(y^h) - C(y^h, \theta) - \lambda S^h(y^h) \\ &= (1 + \lambda)[\tilde{N}(y^h) - C(y^h, \theta)] - \lambda \pi^h(S^h, y^h, \theta). \end{aligned} \quad (28)$$

This utility function consists of two parts. The first part is a linear function (with a positive coefficient) of what we may call the *virtual social benefit*:  $\tilde{N}(y^h) - C(y^h, \theta)$ ; the second part is a linear function of the agent's profit [with

<sup>8</sup>View the model (with  $\lambda = 0$ ) as a revelation game played by two players, the consumers and the agent, concerning their preference for a public good.  $y^*(L)$  [respectively,  $y^*(H)$ ] is the optimal output decision (concerning the public good production  $y \in Y$ ) that maximizes the sum of the consumers' valuation  $N(\cdot)$  of  $y$  (which is public knowledge) plus the agent's reported valuation  $-C(\cdot, L)$  [respectively,  $-C(\cdot, H)$ ] of  $y$ . It is easy to verify that the optimal target-incentive scheme derived from (21), (24), and (25) fits well with a Groves transfer rule. See Green and Laffont (1977).

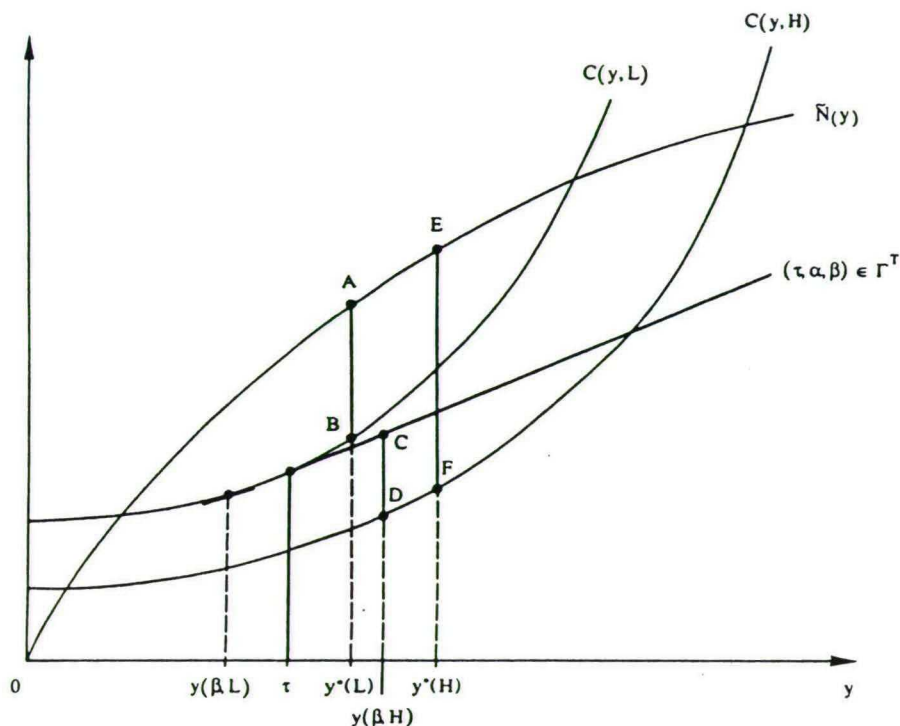


Fig. 3

negative coefficient  $(-\lambda)]$ . When  $\theta$  is known, the optimal output,  $y^*(\theta)$ , maximizes the virtual social benefit (i.e. the distance between  $A$  and  $B$  for  $\theta=L$  and between  $E$  and  $F$  for  $\theta=H$ ) and at this level the agent gets zero profit. When  $\theta$  is unknown, however, the principal is torn between maximizing the expected virtual social benefit and minimizing the agent's profit (subject to non-negative profit constraints). Agent  $L$ 's profit can always be put at zero, but there is a probability of  $1-v$  that the agent is of type  $H$  and gets strictly positive profit (or rents), measured by the distance between  $C$  and  $D$ . To reduce these rents, some virtual social benefit has to be sacrificed, which leads to suboptimal levels of production:  $\tau < y^*(L)$  and  $y(\beta, H) < y^*(H)$ .

### 3.2. Optimal solutions to $P^P$ in the PIS

Let  $(a, b)$  denote a price-incentive scheme:  $S^P(y) = a + by \in \Gamma^P$ . Agent  $\theta$

chooses the output  $y^P(S^P, \theta) = y(b, \theta)$  according to the profit-maximizing condition

$$C_y(y(b, \theta), \theta) = b, \quad \theta = L, H, \quad (29)$$

provided the resulting profit is non-negative. By (29),  $y'(b, \theta) > 0$  and  $y(b, H) > y(b, L)$ . Parallel to (21), the lump-sum payment should be set such that

$$a = C(y(b, L), L) - by(b, L). \quad (30)$$

Inserting (30) into (16) and (17) gives the principal's problem in the PIS as

$$\begin{aligned} \max_b E(W^P | b, v) \stackrel{\text{def}}{=} & v[Ny(b, L)] - (1 + \lambda)C(y(b, L), L) \\ & + (1 - v)[N(y(b, H)) - C(y(b, H), H) \\ & - \lambda(C(y(b, L), L) + b(y(b, H) - y(b, L))]. \end{aligned} \quad (31)$$

*Proposition 3.* For  $v \in (v^P, 1)$  there exists a unique price-incentive scheme  $(a(v), b(v))$  which solves the program (31); it is differentiable on  $(v^P, 1)$  and satisfies (30) and

$$\begin{aligned} v(\tilde{N}'(y(b, L)) - b)y'(b, L) + (1 - v)(\tilde{N}'(y(b, H)) - b)y'(b, H) \\ = \frac{\lambda}{1 + \lambda}(1 - v)(y(b, H) - y(b, L)). \end{aligned} \quad (32)$$

*Proof.* Similar to Proposition 1, and hence is omitted.

It is easily seen that for  $v < 1$ ,

$$\tilde{N}'(y(b, L)) > b, \quad (33)$$

because otherwise the signs on each side of the eq. (32) would be incompatible. This implies that the marginal cost of production is strictly smaller than the marginal virtual payoff for agent  $L$  in the PIS as well. By Propositions 1 and 3, we may denote respectively by  $S^P(v)$  [or  $(a(v), b(v))$ ] and  $S^T(v)$  [or  $(\tau(v), \alpha(v), \beta(v))$ ] the optimal solutions to  $P^T$  and  $P^P$  defined on  $(v^h, 1)$ .

Recall that so far we have been neglecting the possibility of agent  $L$  quitting. In that case the principal can obtain expected welfare  $(1 - v)W^*(H)$ . It is easy to verify that the function  $\zeta^h(v) \stackrel{\text{def}}{=} (1 - v)W^*(H) - E(W^h | S^h(v), v)$  is strictly decreasing in  $v$  (and also continuous), which becomes positive for  $v$



sufficiently small and negative for  $v$  sufficiently large. Thus for  $h = P, T$ , there exists a cut-off belief  $\underline{v}^h$  which is determined by

$$\zeta^h(\underline{v}^h) = 0, \quad (34)$$

such that for  $v > \underline{v}^h$  the solution to  $P^h$  is optimal and for  $v \leq \underline{v}^h$  the optimal scheme is simply  $S^{*h}(H)$ .

### 3.3. Preliminary comparative results

Let  $\pi^h(v)$  denote agent  $H$ 's maximum profit under the optimal incentive scheme:  $\pi^h(v) = 0$  for  $v \leq \underline{v}^h$ , and for  $v > \underline{v}^h$

$$\begin{aligned} \pi^P(v) = & C(y(b(v), L), L) + b(v)[y(b(v), H) - y(b(v), L)] \\ & - C(y(b(v), H), H), \end{aligned} \quad (35)$$

$$\pi^T(v) = C(\tau(v), L) + \beta(v)[y(\beta(v), H) - \tau(v)] - C(y(\beta(v), H), H). \quad (36)$$

Since in the final period of the dynamic relationship the remaining planning problem reduces to a static problem, given the principal's belief about the distribution of  $\theta$  in that period, and the last-period belief may in turn influence the decisions in the previous period of each party, it is of crucial importance to understand how relevant single-period variables and utilities would change with this belief.

*Lemma 2.* For  $v \in (\underline{v}^h, 1)$ ,

- (i)  $\tau'(v) > 0$  and  $\beta'(v) > 0$ ,
- (ii)  $b'(v) > 0$ .

That is, at the optimal solution to  $P^h$ , the target and piece-reward in the TIS and the price in the PIS all increase with the probability of agent  $L$ 's occurrence.

*Proof.* See the appendix.

*Lemma 3.* For  $v \in (\underline{v}^h, 1)$ ,  $\pi^{h'}(v) > 0$ .

*Proof.* From Lemma 2(ii),

$$\pi^{P'}(v) = b'(v)[y(b(v), H) - y(b(v), L)] > 0,$$

and from Lemma 2(i) and Proposition 2,

$$\pi^T(v) = [C_y(\tau, L) - \beta(v)]\tau'(v) + (y(\beta, H) - \tau)\beta'(v) > 0.$$

Q.E.D.

The above lemmas are extensions of similar results in Freixas et al. (1985). As they recognize, the result that agent  $H$ 's profit strictly increases with the principal's probability assessment of  $\theta = L$  provides the very reason why agent  $H$  tends to mimic agent  $L$ . Another useful result is given by

*Lemma 4.* For  $v \in (\underline{v}, 1)$ ,  $\beta(v) < b(v)$ .

*Proof.* See the appendix.

For later use let  $\pi^P(1^-) \stackrel{\text{def}}{=} \lim_{v \rightarrow 1} \pi^P(v)$  and  $\pi^T(1^-) \stackrel{\text{def}}{=} \lim_{v \rightarrow 1} \pi^T(v)$ . What is vital for our comparative analysis is agent  $H$ 's relative levels of utilities in the TIS and the PIS. We will show below (Proposition 10) that in quite general situations  $\pi^T(v) \leq \pi^P(v)$  for  $v > \underline{v}$  with  $\underline{v}$  arbitrarily given. A more immediate result is

*Lemma 5.* Let  $s = \tilde{N}'(y^*(L))$ .

- (i)  $\lim_{v \rightarrow 1} \beta(v) = \beta^+ < s$ .
- (ii) There exists  $\underline{v}^* < 1$  such that for  $v \in (\underline{v}, 1) \cap (\underline{v}^*, 1)$ ,  $\pi^T(v) \leq \pi^P(v)$ .

*Proof.* (i) First note that by taking the limit in (24),  $\lim_{v \rightarrow 1} \tau(v) = y^*(L)$ . From Proposition 2 and Lemma 2 there exists a limit  $\beta^+$  such that  $\beta^+ = \lim_{v \rightarrow 1} \beta(v) \leq \lim_{v \rightarrow 1} C_y(\tau(v), L) = s$ . Thus, we only need to show that  $\beta^+ \neq s$ . Suppose, on the contrary, that  $\beta^+ = s$ . By the assumption that  $C_{yy} > 0$ ,  $y'(\beta, H) = 1/C_{yy}$  is different from infinity for all  $\beta \in R$ . Now take the limit in (25) as  $\beta$  goes to  $s$ . The right-hand side of (25) would be strictly positive while the left-hand side of (25) would be (weakly) negative, which is a contradiction. Therefore  $\beta^+ < s$ .

(ii) Taking limits in (35) and (36), and by the mean value theorem and  $\beta^+ < s$ , we have

$$\begin{aligned} \pi^P(1^-) - \pi^T(1^-) &= s[y(s, H) - y^*(L)] - [\beta^+(y(\beta^+, H) - y^*(L)) \\ &\quad + k(y(s, H) - y(\beta^+, H))] \quad (\beta^+ < k < s) \\ &> (s - k)(y(s, H) - y^*(L)) > 0. \end{aligned}$$

Since  $\pi^h(v)$ ,  $h = P, T$ , are continuous on  $(\underline{v}, 1)$  there must exist an  $\underline{v}^* < 1$  such that  $\pi^T \leq \pi^P$  on  $(\underline{v}, 1) \cap (\underline{v}^*, 1)$ . Q.E.D.

From Lemmas 4 and 5 the piece-reward in the TIS is always strictly smaller than the optimal price for  $v \in (\underline{v}, 1]$ . This is an interesting property.

Note that if  $v=1$ , the optimal incentive scheme is such that  $\beta(1)=\beta^*(L)\leq s$  [see (14)], and  $\beta(1)$  can be equal to  $s$ . Put differently, there is only one 'stable' optimal piece-reward at  $v=1$  which equals  $\beta^*$ ; other piece-reward levels,  $\beta^*(L)\neq\beta^+$ , though optimal at  $v=1$ , become suboptimal and will jump to  $\beta^+$  if we allow estimation errors on  $v$ .

The next proposition shows that the single-period expected welfare in the TIS is strictly higher than that in the PIS.

*Proposition 4.*  $v^T < v^P$  and for  $v \in [0, 1]$ ,

$$E(W^T | (\tau(v), \alpha(v), \beta(v)), v) \geq E(W^P | (a(v), b(v)), v) \quad (37)$$

with strict inequality for  $v \in (v^T, 1)$ .

*Proof.* Suppose first that  $v \in (v^T, 1)$ , and let  $(a, b)$  be the optimal solution to  $P^P$ . Consider a target-incentive scheme  $(\tau, \alpha, \beta)$  such that  $\tau = y(b, L)$ ,  $\alpha = a + by(b, L)$ , and  $\beta = b$ . This  $(\tau, \alpha, \beta)$  is merely a duplicate of  $(a, b)$  for  $y \geq y(b, L)$ , hence

$$E(W^T | (\tau, \alpha, \beta), v) = E(W^P | (a, b), v). \quad (38)$$

But with  $(\tau, \alpha, \beta)$  so defined, we have  $C_y(\tau, L) = \beta$ , and by Proposition 2(ii) it is not a solution to  $P^T$ . Thus, for a target-incentive scheme  $(\tau, \alpha, \beta)$  to be optimal for  $P^T$ , (37) must hold with strict inequality. It follows immediately that  $v^T < v^P$  [see (34)] and thus the strict inequality holds for  $v \in (v^T, 1)$ . In the other cases the expected welfare levels in both systems are equal. Q.E.D.

In the above proof we see that given an optimal price-incentive scheme  $(a, b)$ , there is an equivalent target-incentive scheme which achieves the same result as in  $(a, b)$  with the target  $\tau$  equal to  $y(b, L)$ . This target-incentive scheme is not optimal in the TIS. There are two possible ways to improve it: either by increasing the target  $\tau$  from the level of  $y(b, L)$  while keeping the slope  $\beta = b$  constant, or by reducing the slope  $\beta$  from the level of  $b$  while keeping  $\tau = y(b, L)$ . For more intuition consider the latter case (see fig. 4), which is likely to happen if  $v$  is large. In the  $(a, b)$ , the curve passing through  $B$  and  $F$  is agent  $H$ 's indifference curve. If we reduce the slope from the level of  $b$  to  $\beta$ , agent  $H$  will reduce production by producing  $y(\beta, H) < y(b, H)$ , and suffer a reduction of profit measured by the distance between  $B$  and  $C$ . We may perceive the output reduction as a 'substitution effect': the change along the same indifference curve of agent  $H$  from  $F$  to  $B$ , and the profit reduction as an 'income effect': the vertical change from  $B$  to  $C$ . When  $v$  is close to 1, the shift from  $F$  to  $C$  has no effect in state  $L$  and has (very) little effect in state  $H$  on the virtual social benefit [compare the distance between  $E$  and  $G$

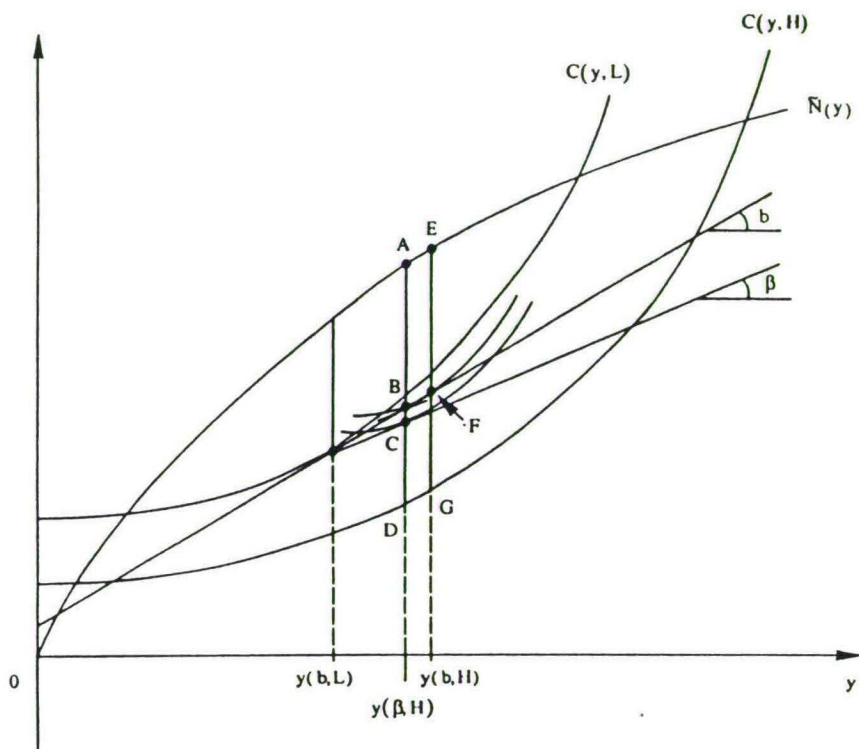


Fig. 4

and between  $A$  and  $D$ , see also Lemma 4(iii)], but it has more significant effect on agent  $H$ 's profit. By (28), this reduction of price ( $\beta$  instead of  $b$ ) improves welfare.

#### 4. Incentive systems in a two-period relationship with incomplete information

##### 4.1. Dynamic game and continuation equilibrium

We follow the existing literature to model the problem as a sequential game with incomplete information. The game contains five stages (see fig. 5). In stage zero, one of the incentive systems of TIS and PIS,  $\Gamma^h$ , is established and fixed for both periods. In stage one, Nature chooses  $\theta$  with  $\text{Prob}\{\theta=L\}=v_1$  and  $\text{Prob}\{\theta=H\}=1-v_1$ . While this probability is common knowledge, only the agent is informed of the true value of  $\theta$ . In stage two, the principal chooses a first-period incentive scheme,  $S_1^h$ , from  $\Gamma^h$  and offers it to the agent. In stage three the agent produces  $y_1^h$  in response to  $S_1^h$  at cost



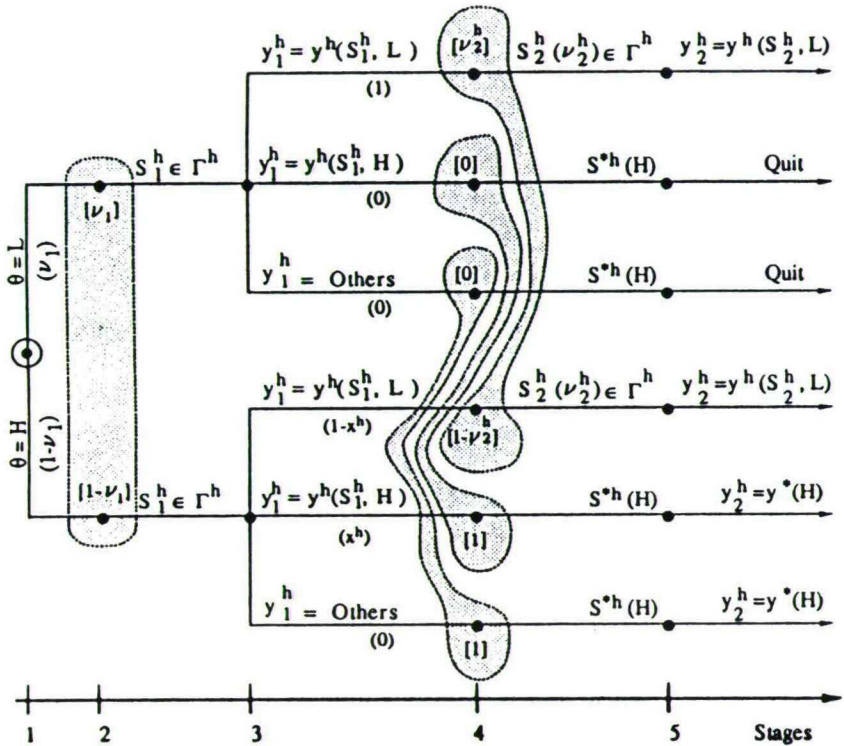


Fig. 5. Game tree highlighting the equilibrium paths. The shaded areas are the information sets. The probabilities of playing the corresponding strategy are in parentheses. The principal's probability assessments of his position in the information set are in square brackets. The term 'others' denotes all the output levels deviating from  $y^h(S_1^h, L)$  and  $y^h(S_1^h, H)$ . The probability of reaching this information set is zero, thus the posterior beliefs in this set can be specified somewhat arbitrarily, provided that they are Bayes consistent [see Kreps and Wilson (1982)].

$C(y_1^h, \theta)$ , receives a reward  $S_1^h(y_1^h)$ . In the following stage the principal chooses another incentive scheme,  $S_2^h$ , from  $\Gamma^h$ . Finally, the agent responds to  $S_2^h$  by producing  $y_2^h$ , and receives the second-period payment,  $S_2^h(y_2^h)$ . Details of the game tree depicted in fig. 5 will be clear shortly.

Similar to the preceding research strategy we assume that there is a 'non-degenerate set',  $V_1^h$ , such that for  $v_1 \in V_1^h$  it is optimal to induce both types of agents to produce, and to shut down agent  $L$  from the outset otherwise.<sup>9</sup> We will show below that  $V_1^h$  is properly defined. Let  $V_1 = V_1^P \cap V_1^T$ .

<sup>9</sup>We are not sure if  $V_1^h$  is an interval as it was in the static case, because knowing the shape of an optimal total discounted expected welfare is difficult without a more elaborate analysis of the PB equilibria. But it does not matter for our analysis.

The two partners' respective payoffs in this dynamic game, i.e. the total discounted profit for the agent and the total discounted welfare for the principal, are given in (5) and (6). The principal's strategy is a sequence of incentive schemes,  $S_1^h$  and  $S_2^h$ ,<sup>10</sup> both selected from the incentive system  $\Gamma^h$ . The strategy of the agent is a sequence of outputs,  $y_1^h$  and  $y_2^h$ , chosen from  $Y$ . Let  $v_2^h$  denote the principal's probability assessment of  $\theta=L$  at the start of the second-period.  $v_2^h$  will depend on  $v_1$ , the agent's first-period strategy and the observed first-period output, as well as the first-period incentive scheme actually adopted.

The notion of perfect Bayesian (PB) equilibria is suitable for analysis. A PB equilibrium in  $\Gamma^h$  is a set of (possibly mixed) strategies and beliefs  $\{S_1^h, y_1^h, S_2^h, y_2^h, v_2^h\}$  satisfying sequential optimality and the Bayes rule [see Freixas et al. (1985) and Laffont and Tirole (1987)]. In the present context, a PB equilibrium satisfies the following five conditions:

C1.  $y_2^h$  is optimal for the agent given  $S_2^h$ .

C2.  $S_2^h$  is optimal for the principal given his belief  $v_2^h$ .

C3.  $y_1^h$  is optimal for the agent given  $S_1^h$  and the fact that the principal's second-period belief depends on the observed first-period output.

C4.  $S_1^h$  is optimal for the principal given the subsequent strategies.

C5.  $v_2^h$  is Bayes-consistent with the principal's prior belief  $v_1$  and the agent's first-period strategy and the observed first-period output.

The derivation of a PB equilibrium is analytically complicated. But since the comparative analysis does not hinge upon explicit calculations of PB equilibria, the major part of the analysis can be carried out using a less stringent notion of equilibrium, namely the *continuation equilibrium*. A continuation equilibrium is defined as a set of strategies and beliefs satisfying the conditions for a PB equilibrium except that  $S_1^h$  is arbitrarily given. A *perfect Bayesian equilibrium* is simply an optimal (for the principal) continuation equilibrium. It turns out that the existence of a PB equilibrium holds in both the TIS and the PIS (Proposition 6).

The continuation equilibrium (henceforth often called an equilibrium) has

<sup>10</sup>The arguments of the strategies and beliefs are often omitted for brevity.

been analyzed by Freixas et al. (1985) in a similar context. In what follows we first summarize their results concerning the existence and uniqueness of a continuation equilibrium of the sequential game. It is shown that there exists one and only one continuation equilibrium for any first-period incentive scheme in either incentive system. Consequently, the principal's welfare-maximizing problem is well defined. We will then formulate explicitly the principal's dynamic optimization problem and show that a perfect Bayesian equilibrium exists in either incentive system.<sup>11</sup>

Given  $S_1^h \in \Gamma^h$ , let  $y(S_1^h, \theta)$  denote agent  $\theta$ 's single-period, profit-maximizing output and  $y_1^h(S_1^h, \theta)$  his first-period output taking into account the second-period strategies. A first result related to agent  $L$ 's strategy is that  $y_1^h(S_1^h, L) = y(S_1^h, L)$ . This is because, according to our analysis in the previous section, whatever the principal's second-period belief is, agent  $L$  can never make a profit higher than his reservation level in the second period. Given this, agent  $H$ 's first-period strategy should be either  $y_1^h(S_1^h, H) = y(S_1^h, L)$  or  $y_1^h(S_1^h, H) = y(S_1^h, H)$ . Indeed, if a level of first-period output other than  $y(S_1^h, L)$  is observed, from agent  $L$ 's strategy and the Bayes rule, the principal's posterior belief about the agent's type will be  $\text{Prob}\{\theta = L\} = 0$ , which implies that agent  $H$  will be offered  $S_2^h = S^{*h}(H)$  and put at his zero reservation utility level in the second period. Therefore  $y_1^h(S_1^h, H)$  ought to maximize agent  $H$ 's first-period profit, i.e. it must be  $y(S_1^h, H)$ , unless the agent chooses to produce  $y(S_1^h, L)$ .

As a result, in a continuation equilibrium agent  $L$  produces  $y(S_1^h, L)$ , and agent  $H$  assigns a probability  $x^h \in [0, 1]$  to producing  $y(S_1^h, H)$  and  $1 - x^h$  to producing  $y(S_1^h, L)$ . By the Bayes rule the principal's second-period probability assessment of  $\theta = L$  is  $v_2^h(S_1^h, v_1, y(S_1^h, H)) = 0$  if he observes  $y(S_1^h, H)$  and  $v_2^h(S_1^h, v_1, y(S_1^h, L)) = v_1 / (v_1 + (1 - v_1)(1 - x^h))$  if he observes  $y(S_1^h, L)$ . We would say that a continuation equilibrium is pooling if  $x^h = 0$ , separating if  $x^h = 1$ , and semi-separating if  $x^h \in (0, 1)$ .

If agent  $H$  chooses to produce  $y(S_1^h, L)$  in the first period, some short-run profit, denoted  $\Omega^h(S_1^h)$ , is forgone. It is given by

$$\Omega^h(S_1^h) = \pi_1^h(S_1^h, y(S_1^h, H), H) - \pi_1^h(S_1^h, y(S_1^h, L), H). \quad (39)$$

Agent  $H$  may also expect a gain in the second period by choosing  $y(S_1^h, L)$ , the present value of which is  $\delta\pi(v_2^h)$ . Comparing  $\Omega^h(S_1^h)$  with  $\delta\pi(v_2^h)$  gives:

*Proposition 5. For any  $S_1^h \in \Gamma^h$ , there exists one and only one continuation equilibrium the type of which is determined as follows:*

<sup>11</sup>There may be multiple perfect Bayesian equilibria, but it does not pose a problem here since the principal moves first and each first-period incentive scheme leads to a unique continuation equilibrium.



(i) It is a pooling equilibrium if and only if

$$\Omega^h(S_1^h) \leq \delta\pi^h(v_1). \quad (40)$$

(ii) It is a separating equilibrium if and only if

$$\Omega^h(S_1^h) \geq \delta\pi^h(1^-). \quad (41)$$

(iii) It is a semi-separating equilibrium if and only if

$$\delta\pi^h(v_1) < \Omega^h(S_1^h) < \delta\pi^h(1^-). \quad (42)$$

*Proof.* See Freixas et al. (1985, Proposition 4).

Since  $\pi^h(v)$  is continuous and strictly increasing in  $v$ , condition (42) is equivalent to saying that there exists a  $v_2^h \in (v_1, 1)$  such that

$$\Omega^h(S_1^h) = \delta\pi(v_2^h). \quad (43)$$

The one-to-one relationship between  $v_2^h$  and  $x^h$  is given by

$$v_2^h = v_1 / (v_1 + (1 - v_1)(1 - x^h)). \quad (44)$$

Recall that  $v_2^h$  is the principal's posterior belief conditional on the observation of  $y(S_1^h, L)$  given agent  $H$ 's first-period strategy  $x^h$  [ $x^h = 0$  corresponds to (40) and  $x^h = 1$  to (41)]. By Proposition 5, for any given  $S_1^h$  there is a unique first-period equilibrium strategy completely given by  $x^h \in [0, 1]$  for agent  $H$ , which is derived from conditions (40), (41), (43), and (44) via  $v_2^h$ .

#### 4.2. Perfect Bayesian equilibrium

Given  $S_1^h$ , and hence  $x^h$ , the probability of observing  $y(S_1^h, L)$  in the first period is  $v_1 + (1 - v_1)(1 - x^h)$ . The probability of observing  $y(S_1^h, H)$  is  $(1 - v_1)x^h$ . When  $y(S_1^h, L)$  is observed, the second-period expected welfare will be  $E(W^h | S_2^h, v_2^h)$  as defined in (15). Let  $W^h(v_2^h)$  denote  $\max_{S_2^h} E(W^h | S_2^h, v_2^h)$  subject to the agent's non-negative profit constraint [as in (17)]. The a priori expected second-period welfare, denoted  $W_2^h$ , can then be written as

$$W_2^h = (v_1 + (1 - v_1)(1 - x^h)) W^h(v_2^h) + (1 - v_1)x^h W^*(H). \quad (45)$$

Notice that the first-period lump-sum transfer of funds,  $a_1$  or  $\alpha_1$ , does not appear in  $\Omega^h(S_1^h)$ , and therefore does not influence the second-period

strategies. Assuming that both agent's rationality constraints should be met, i.e.  $v_1 \in V_1^h$ , we have the binding conditions:

$$a_1 = C(y(b_1, L), L) - b_1 y(b_1, L) \quad (46)$$

and

$$\alpha_1 = C(\tau_1, L). \quad (47)$$

These equations define  $a_1$  and  $\alpha_1$  as functions of  $b_1$  and  $(\tau_1, \beta_1)$ , respectively. Thus, we only need to consider the principal's first-period decision variables,  $b_1$  and  $(\tau_1, \beta_1)$ , respectively. The following is an analogous result to that given in Lemma 1.

*Lemma 6.* *There is no loss of generality to restrict attention to  $C_y(\tau_1, H) \leq \beta_1 \leq C_y(\tau_1, L)$  in search of an optimal  $(\alpha_1, \tau_1, \beta_1)$ .*

*Proof.* Similar to Lemma 1, and hence is omitted; the only additional consideration is that of the influence of this restriction on  $\Omega^h(S_1^h)$ .

By Lemma 6 we may write  $y(S_1^T, L) = \tau_1$  and  $y(S_1^T, H) = y(\beta_1, H)$ . As to the price-incentive scheme, we always have  $y(S_1^P, L) = y(b_1, L)$  and  $y(S_1^P, H) = y(b_1, H)$ . The cost of concealing information for agent  $H$ ,  $\Omega^h(S_1^h)$ , can now be more explicitly expressed as

$$\begin{aligned} \Omega^P(S_1^P) &= \Omega^P(b_1) \\ &= b_1(y(b_1, H) - y(b_1, L)) - [C(y(b_1, H), H) - C(y(b_1, L), H)], \end{aligned} \quad (48)$$

$$\begin{aligned} \Omega^T(S_1^T) &= \Omega^T(\tau_1, \beta_1) \\ &= \beta_1(y(\beta_1, H) - \tau_1) - [C(y(\beta_1, H), H) - C(\tau_1, H)]. \end{aligned} \quad (49)$$

Let us state a simple but important fact before writing out the welfare functions in detail.

*Lemma 7.* *In the TIS (PIS), it is never optimal to induce a pooling or semi-separating equilibrium such that  $v_2 \leq \underline{v}^T$  ( $v_2 \leq \underline{v}^P$ ).*

*Proof.* Because  $v_1 \leq v_2 \leq \underline{v}^h$ , and given these beliefs, the separating scheme,  $S^{*h}(H)$ , which forces agent  $L$  to quit is superior to any other schemes in both periods [see (45) and use  $W^h(\underline{v}_2^h) \leq (1 - \underline{v}_2^h)W^*(H)$ ]. Q.E.D.

This lemma implies that if agent  $L$  does not quit in the first period he

should not be induced to quit in the second period. For the moment assume  $v_1 \in V_1$ . By Lemma 7 we can write

$$W_1^P(L) \stackrel{\text{def}}{=} N(y(b_1, L)) - (1 + \lambda)C(y(b_1, L), L), \quad (50)$$

$$W_1^T(L) \stackrel{\text{def}}{=} N(\tau_1) - (1 + \lambda)C(\tau_1, L), \quad (51)$$

i.e. the first-period welfare when  $\theta = L$ ;

$$W_1^P(L/H) \stackrel{\text{def}}{=} N(y(b_1, L)) - C(y(b_1, L), H) - \lambda C(y(b_1, L), L) \quad (52)$$

$$W_1^T(L/H) \stackrel{\text{def}}{=} N(\tau_1) - C(\tau_1, H) - \lambda C(\tau_1, L), \quad (53)$$

i.e. the first-period welfare when  $\theta = H$  and agent  $H$  produces  $y(b_1, L)$  under  $\Gamma^P$  and  $\tau_1$  under  $\Gamma^T$  in the first period;

$$\begin{aligned} W_1^P(H) \stackrel{\text{def}}{=} & N(y(b_1, H)) - C(y(b_1, H), H) \\ & - \lambda [C(y(b_1, L) + b_1(y(b_1, H) - y(b_1, L))), \end{aligned} \quad (54)$$

$$\begin{aligned} W_1^T(H) \stackrel{\text{def}}{=} & N(y(\beta_1, H)) - C(y(\beta_1, H), H) \\ & - \lambda [C(\tau_1, L) + \beta_1(y(\beta_1, H) - \tau_1)], \end{aligned} \quad (55)$$

i.e. the first-period welfare when  $\theta = H$  and agent  $H$  produces  $y(b_1, H)$  or  $y(\beta_1, H)$  in the first period.

The first-period expected welfare, given  $S_1^h$  and the subsequent equilibrium strategies, can be written as

$$W_1^h = v_1 W_1^h(L) + (1 - v_1)(1 - x^h) W_1^h(L/H) + (1 - v_1)x^h W_1^h(H). \quad (56)$$

Finally, the total discounted expected welfare in each of the two systems is<sup>12</sup>

$$E(W^P, x^P | b_1, v_1) = W_1^P + \delta W_2^P, \quad (57)$$

$$E(W^T, x^T | (\tau_1, \beta_1), v_1) = W_1^T + \delta W_2^T. \quad (58)$$

The principal's first-period optimization problem is either to choose  $b_1$  to maximize (57) in the PIS, or to choose the target-bonus pair  $(\tau_1, \beta_1)$  to

<sup>12</sup>The term  $x^h$  need not be made explicit in (57) and (58) in equilibria since it is determined once  $S_1^h$  is given. However, it is useful for consequent analyses.

maximize (58) [subject to  $C(\tau_1, L) \geq \beta_1 \geq C(\tau_1, H)$ ] in the TIS. It is easy to obtain

*Proposition 6.* (i) For  $v_1 \in V_1$ ,  $E(W^P, x^P | b_1, v_1)$  and  $E(W^T, x^T | (\tau_1, \beta_1), v_1)$  are continuous in  $b_1$  and in  $(\tau_1, \beta_1)$  respectively. Therefore

(ii) A perfect Bayesian equilibrium exists under both PIS and TIS.

*Proof.* See the appendix. The proof amounts to a straightforward check of continuity at  $x^h=0$  and  $x^h=1$ , and of the fact that the total discounted expected welfare declines when  $b_1$  or  $\tau_1$  is sufficiently large.

By this proposition, the optimal total discounted expected welfare is well defined as a function of  $v_1$ . It is given by  $E(W^h, x^h | S_1(v_1), v_1)$  [see (57) and (58)]. Let  $\zeta_1^h(v_1) \triangleq (1-v_1)(1+\delta)W^*(H) - E(W^h, x^h | S_1(v_1), v_1)$ . The non-degenerate set  $V_1^h$  can now be defined as  $V_1^h \triangleq \{v_1 \in (0, 1) | \zeta_1^h(v_1) < 0\}$ . This set is not empty because  $\zeta_1^h(v_1) < 0$  for  $v_1$  sufficiently close to 1.

#### 4.3. TIS vs. PIS

We now proceed to compare the TIS with the PIS in the dynamic setting. We first show that given a first-period incentive scheme, agent  $H$  assigns higher probability to revealing his private information if the precommitted second-period incentive system is the TIS rather than PIS, provided that  $v_1$  is high enough. We then prove that for a given first-period incentive scheme a higher probability of truth revelation entails a higher total discounted expected welfare. These properties imply that as long as  $v_1$  is high enough the TIS dominates the PIS in achieving a higher discounted welfare. Finally, under further restrictions on the benefit and cost functions we show that for all  $v \in (v, 1)$ ,  $\pi^T \leq \pi^P$ , and hence the TIS dominates the PIS for all  $v_1 \in [0, 1]$  for the specified class of benefit and cost functions.

*Proposition 7.* Suppose  $v_1 \in V_1 \cap (v^*, 1)$ . Given an arbitrary first-period incentive scheme  $S_1$ , let  $x^P$  ( $x^T$ ) denote the corresponding strategies of agent  $H$  when the principal precommits to PIS (TIS) for the second period. We have  $x^T \geq x^P$ ; and if  $x^P \in (0, 1)$ ,  $x^T > x^P$ .

*Proof.* See the appendix.

Note that in Proposition 7 we use a common first-period incentive scheme for both systems. This allows us to focus on the relative ratchet-effect implications of different commitment policies w.r.t. future periods. In this context, the value of  $1 - x^h$  is the legitimate indicator of the seriousness of the ratchet effect since it is linearly and positively related to the probability that



a first-period incentive scheme does not attain a separation [see (45)]. Using this indicator we have shown in Proposition 7 that committing to TIS in the second period always mitigates the ratchet effect, provided that  $v_1 \in V_1$  and  $v_1 > v_2^*$ .

If we substitute  $x$  for  $x^h$  and  $v_2 = v_1/(v_1 + (1 - v_1)(1 - x))$  for  $v_2^h$  into (57) and (58), for a fixed first-period incentive scheme, the total discounted expected welfare may be viewed as a function of  $x$ . Denote this function by  $E(W^P, x | b_1, v_1)$  and  $E(W^T, x | (\tau_1, \beta_1), v_1)$ , respectively.

*Proposition 8.* For  $v_1 \in V_1^T$  ( $v_1 \in V_1^P$ ) and a given first-period incentive scheme  $(\tau_1, \alpha_1, \beta_1)$  [ $(a_1, b_1)$ ], if  $\beta_1 \leq \tilde{N}'(y^*(H))$  [ $b_1 \leq \tilde{N}'(y^*(H))$ ], then

$$\frac{dE(W^T, x | (\tau_1, \beta_1), v_1)}{dx} > 0 \left[ \frac{dE(W^P, x | b_1, v_1)}{dx} > 0 \right].$$

*Proof.* See the appendix.

Proposition 8 suggests that the total discounted expected welfare increases with the probability of separation. It confirms the undesirability of the ratchet effect. Another useful observation is given by

*Lemma 8.* For  $v_1 \in V_1$ , and a first-period price-incentive scheme  $(a_1, b_1)$ , let  $r = \min \{b_1, \tilde{N}'(y^*(H))\}$ . If a first-period target-incentive scheme  $(\hat{\tau}_1, \hat{\alpha}_1, \hat{\beta}_1)$  satisfies

$$\hat{\tau}_1 = y(b_1, L), \quad \hat{\alpha}_1 = a_1 + b_1 y(b_1, L), \quad \hat{\beta}_1 \in [r, b_1], \quad (59)$$

then

$$E(W^T, x | (\hat{\tau}_1, \hat{\beta}_1), v_1) \geq E(W^P, x | b_1, v_1),$$

with strict inequality for  $x < 1$ .

*Proof.* See the appendix.

We eventually come to

*Proposition 9.* Suppose  $v_1 \in (v_2^*, 1)$ . Then

$$E(W^T, x^T | (\tau_1(v_1), \beta_1(v_1)), v_1) \geq E(W^P, x^P | b_1(v_1), v_1), \quad (60)$$

with strict inequality for  $v_1 \in V_1^T$ .

*Proof.* See the appendix.

Admittedly, the above result depends on an endogenous assumption that  $v_1 \in (\underline{v}^*, 1)$ , which is not satisfactory. We need more restrictions on the benefit and cost functions to show the general dominance of the TIS over PIS.

*Assumption 3.*  $\tilde{N}'(y) \equiv s$ ;

*Assumption 4.*  $C_{yy}(y, H) \geq \lambda M C_{yyy}(y, H)$  for all  $y \leq y(s, H)$ , where  $M = \max_{x \leq s} |y(x, H) - y(x, L)|$ ,  $y(\cdot, \theta)$  being the inverse function of  $C_y(\cdot, \theta)$ .

Assumption 3 is the case considered in Freixas et al. (1985). It may be justified by regarding the output  $y$  as a money return and assume that the principal is a risk-neutral wealth maximizer. Assumption 4 poses a further restriction on the cost structures. But note that it is still general enough to encompass a large class of situations. For instance, it is always satisfied when  $C_{yyy}/C_{yy}$  is small, or when  $\lambda$  is small.

*Proposition 10.* Under Assumptions 3 and 4 (i)  $\pi^T(v) \leq \pi^P(v)$  for all  $v \in (\underline{v}, 1)$ ; thus (ii) for all  $v_1 \in [0, 1]$

$$E(W^T, x^T | (\tau_1(v_1), \beta_1(v_1), v_1)) \geq E(W^P, x^P | b_1(v_1), v_1), \quad (61)$$

with strict inequality for  $v_1 \in V^T$ .

*Proof.* (i) Write  $\phi = (1 + \lambda)v/(\lambda + v)$  and  $b_\tau = \phi s + (1 - \phi)\beta$ . The proof is by contradiction. Suppose for some  $v \in (\underline{v}, 1)$ ,  $\pi^T(v) > \pi^P(v)$ . We must have  $\tau > y(b, L)$  and  $b_\tau > b$  for otherwise  $(\tau, \alpha, \beta)$  will be below  $(a, b)$  for all the values of  $y$ , which implies  $\pi^T \leq \pi^P$ . From the definition of  $\pi^h$  it follows that

$$\begin{aligned} & C(\tau, L) - C(y(b, L), L) + b[y(\beta, H) - y(b, H) - (\tau - y(b, L))] \\ & + (\beta - b)(y(\beta, H) - \tau) - [C(y(\beta, H), H) - C(y(b, H), H)] > 0. \end{aligned} \quad (62)$$

Let  $D(x) = C(y(x, L), L) - by(x, L)$ ,  $D'(b) = 0$ . By Taylor's expansion, and using the facts  $C_{yy}(y(x, \theta), \theta)y'(x, \theta) = 1$  and  $y'' \leq 0$ ,

$$\begin{aligned} D(b_\tau) - D(b) &= D''(\xi) \frac{(b_\tau - b)^2}{2} \\ &= C_{yy}(y(\xi, L), L)y'^2(\xi, L) \frac{(b_\tau - b)^2}{2} \end{aligned}$$

$$\begin{aligned}
& + [C_y(y(\xi, L), L) - b] y''(\xi, L) \frac{(b_\tau - b)^2}{2} \\
& \leq y'(b, L) \frac{(b_\tau - b)^2}{2} \quad (b \leq \xi \leq b_\tau).
\end{aligned}$$

Similarly, we have

$$\begin{aligned}
& b[y(\beta, H) - y(b, H)] - [C(y(\beta, H), H) - C(y(b, H), H)] \\
& \leq -y'(b, H) \frac{(b - \beta)^2}{2}.
\end{aligned}$$

Thus, from (62) it follows that

$$y'(b, L)[\phi(s - \beta) - (b - \beta)]^2 - y'(b, H)(b - \beta)^2 \geq 2[y(\beta, H) - \tau](b - \beta). \quad (63)$$

Substituting (25) into (63) and manipulating terms yields:

$$\begin{aligned}
& y'(b, L)\lambda(\phi(s - \beta))^2 - 2y'(b, L)\lambda\phi(s - \beta)(b - \beta) \\
& \geq (b - \beta)^2[(1 + 2\lambda)y'(b, H) - \lambda y'(b, L)] \\
& \quad + (1 + \lambda)(b - \beta)(2s - b - \beta)y'(b, H) \\
& \quad - 2(1 + \lambda)(s - \beta)(b - \beta)[y'(\beta, H) - y'(b, H)], \quad (64)
\end{aligned}$$

which, by Assumption 1 and the fact that  $s - b > (1 - \phi)(s - \beta)$ , implies

$$\begin{aligned}
& y'(b, L)\phi(s - \beta)v \geq (b - \beta) \left[ 2vy'(b, L) + \frac{(\lambda + v)}{\lambda}(2 - \phi)y'(b, H) \right] \\
& \quad + 2\frac{(\lambda + v)}{\lambda}(b - \beta)[y'(\beta, H) - y'(b, H)]. \quad (65)
\end{aligned}$$

On the other hand, recalling that  $\tau = y(b_\tau, L)$ , from (25) we have

$$\begin{aligned}
& (s - \beta)(1 - v)y'(\beta, H) \\
& = \frac{\lambda(1 - v)}{(1 + \lambda)} [y(\beta, H) - y(b, L) - (\tau - y(b, L))]
\end{aligned}$$



$$\geq \frac{\lambda(1-v)}{(1+\lambda)} [y(\beta, H) - y(b, L) - y'(b, L)\phi(s-\beta)]. \quad (66)$$

Subtracting (32) from (66) and manipulating terms yields:

$$\begin{aligned} & y'(b, L)\phi(s-\beta)v \\ & \leq (b-\beta) \left[ vy'(b, L) + (1-v) \frac{1+2\lambda}{1+\lambda} y'(b, H) \right] \\ & \quad + (1-v) \left[ (s-\beta) + (b-\beta) \frac{\lambda}{1+\lambda} \right] [y'(\beta, H) - y'(b, H)] \\ & = (b-\beta) \left[ vy'(b, L) + (1-v) \frac{1+2\lambda}{1+\lambda} y'(b, H) \right] \\ & \quad - (1-v) \left[ (s-b) + (b-\beta) \frac{1+2\lambda}{1+\lambda} \right] y''(\eta, H) \quad (\beta \leq \eta \leq b). \quad (67) \end{aligned}$$

Comparing (65) with (67) we derive

$$vy'(b, L) + \frac{1-v}{1+\lambda} y'(b, H) < -(1-v)(s-b)y''(\eta, H). \quad (68)$$

Notice that  $(1-v)y'(\beta, H) \leq vy'(b, L) + (1-v)y'(b, H)$  [compare (25) and (32)]. Expression (68) thus implies

$$y'(\eta, H) < -(1+\lambda)(s-\eta)y''(\eta, H).$$

Using the facts that  $C_{yyy}(y')^2 + C_{yy}y'' = 0$  and  $(s-\eta)y'(\eta, H) < [\lambda/(1+\lambda)]M$  [see (25)], we must have

$$C_{yy}(y(\eta, H), H) < \lambda MC_{yyy}(y(\eta, H), H).$$

But this is a contradiction to Assumption 4. Thus (i) is proved.

(ii) This is an immediate corollary of Proposition 9 and (i). Q.E.D.

**Remarks.** (1) If different incentive systems are committed to in each period, it is easily seen that choosing the TIS for both periods strictly dominates

choosing the TIS and the PIS in the first period and the second period, respectively, and at least weakly dominates choosing the PIS in the first period and the TIS in the second period.

(2) The proof that the high productivity agent enjoys less information rents in the TIS than in the PIS is unexpectedly complicated (Proposition 10). Perhaps this is why the intuition behind this observation is so hard to get. I find it particularly difficult to conjecture whether this fact stems from the highly stylized model construction, or there is any intrinsic logic lying at the bottom.

(3) On the other hand, even if we could find an instance where the above observation is false (as suggested by Assumptions 3 and 4, such an instance would have to be found with more complex benefit and/or cost functions), we are not sure that the PIS would then outperform the TIS. The dynamic losses of adopting the more general incentive system, if any, would then have to be compared with the likely static gains, and it would be hard to draw a general conclusion.

## 5. Summary and remarks

This paper has undertaken a comparative analysis of incentive systems in the context of a repeated principal-agent relationship with adverse selection and ratchet effects. Two widely used incentive systems, the price-incentive system (PIS) and the target-incentive system (TIS), are analyzed and their associated (total discounted) expected welfare levels attained at the perfect Bayesian equilibrium are compared. The principal is assumed to be committed to a particular incentive system for the whole planning horizon, whereas the parameters of the system, or in other words, the specific schemes, are allowed to be revised periodically. Under such a framework the ratchet effect is evident, but varies with different incentive systems. The comparison is carried out through careful examinations of equilibrium strategies and beliefs under each incentive system. It is found that employing the TIS can lead to a higher probability of early truth revelation than the PIS without creating extra costs for the principal. This fact together with the strict welfare dominance of TIS over PIS in static settings (proved in section 3) allows us to conclude that the TIS also strictly outperforms the PIS in multi-period relationships.

From a positive point of view, this result provides some justification for the frequent use of targets, quotas, etc. for certain (controlled) economic variables in hierarchical organizations, such as in planned economies or multinational private firms. Namely, under asymmetric information, a system that allows the central authority or headquarters only to control transfer prices (with lump-sum transfers) can be improved by allowing them to stipulate minimal required targets as well.

From the normative point of view, our analysis might serve as an initial step towards optimal incentive system design for long-term decentralized planning with periodic scheme revisions. If *scheme* commitment is possible, the problem of optimal system design is trivial since any optimal inter-temporal schemes chosen from a particular incentive system belong also to a more general system, i.e. the more general the system is, the better. If scheme commitment is impossible, then there is a trade-off: on the one hand, given the fact that any dynamic relationship governed by equilibrium short-term schemes can be duplicated by a long-term scheme, the optimum attained at the equilibria with short-term incentive schemes unconstrained by any incentive system<sup>13</sup> might be improved if the principal can precommit to a particular incentive system; on the other hand, if the incentive system is too restrictive, as is the case with the PIS, relaxing commitment can improve welfare. Thus, the interest in designing optimal long-term incentive systems for relationships governed by short-term contracts (schemes) is justified. Existing analyses, in one way or another, have only been comparative, focusing on simplified incentive structures descriptive of actual arrangements, and have not addressed the admittedly difficult problem of optimality. But perhaps a normative theory of optimal incentive system design may emerge through a 'tâtonnement' process of judicious comparative analyses.

A target-incentive scheme can be seen as a piece-linear function with a kink at the target level. A follow-up of this study might be to investigate how piece-linear schemes with two or more kinks work. For instance, given two systems of piece-linear incentive schemes, is it true that the one always outperforms the other if the former allows more kinks for the scheme structure? For such investigations to make sense, we might have to extend the model to allow for more than two possible types of agents. Although analytical difficulties are inevitable, the qualitative result in this paper seems likely to hold as well.

The present model involves only a pure adverse selection type of informational asymmetry, where the agent is assumed to possess complete information concerning the production environment. How results may alter in the presence of output uncertainty and moral hazard, or when there is both moral hazard and adverse selection, seems to be also an interesting as well as promising topic for further research.

## Appendix

*Proof of Proposition 1.* Let  $\bar{W} = E(W^T | (\tau, \beta), v)$ . For  $v \in (v^T, 1)$ , the first-order conditions for an optimal  $(\tau, \beta)$  in (22) are

<sup>13</sup>Laffont and Tirole (1988) have developed techniques for designing optimal sequential incentive schemes in a repeated principal-agent setting without precommitment to an incentive system.



$$\bar{W}_\tau = v(1 + \lambda)[\tilde{N}'(\tau) - C_y(\tau, L)] - (1 - v)\lambda[C_y(\tau, L) - \beta] = 0, \quad (69)$$

$$\bar{W}_\beta = (1 - v)[(1 + \lambda)(\tilde{N}'(y(\beta, H)) - \beta)y'(\beta, H) - \lambda(y(\beta, H) - \tau)] = 0. \quad (70)$$

Conditions (24) and (25) follow immediately from (69) and (70). Furthermore, one may verify that

$$\bar{W}_{\tau\tau} = \lambda(1 + \lambda)[\tilde{N}''(\tau) - C_{yy}(\tau, L)] - \lambda(1 - v)C_{yy}(\tau, L) < 0, \quad (71)$$

$$\begin{aligned} \bar{W}_{\beta\beta} = (1 - v)(1 + \lambda) & \left[ \tilde{N}''(y(\beta, H)) \left( \frac{dy(\beta, H)}{d\beta} \right)^2 \right. \\ & \left. + (\tilde{N}'(y(\beta, H)) - \beta) \frac{d^2 y(\beta, H)}{d\beta^2} - \frac{1 + 2\lambda}{1 + \lambda} y'(\beta, H) \right] < 0, \end{aligned} \quad (72)$$

$$\bar{W}_{\tau\beta} = \lambda(1 - v) > 0. \quad (73)$$

Eqs. (71) and (73) are obvious ( $\tilde{N}'' \leq 0, C_{yy} > 0$ ). Eq. (72) comes from Assumption 1 and condition (20). By Assumptions 1 and 2, the Jacobian of (69) and (70):

$$\begin{aligned} J &= \bar{W}_{\tau\tau} \bar{W}_{\beta\beta} - (\bar{W}_{\tau\beta})^2 \\ &\geq (\lambda + v)C_{yy}(\tau, L)(1 - v)(1 + 2\lambda)y'(\beta, H) - \lambda^2(1 - v)^2 \\ &\geq [(\lambda + v)C_{yy}(y(\beta, L), L)(1 + 2\lambda) - \lambda^2(1 - v)C_{yy}(y(\beta, H), H)] \\ &\quad \times (1 - v)y'(\beta, H) \\ &> 0. \end{aligned} \quad (74)$$

Eqs. (71)–(74) show that the second-order condition for the maximization problem (22) is satisfied. Thus, by the assumption that the production is worth carrying out with both types of agents,  $\tau$  and  $\beta$  exist and are uniquely determined by (69) and (70). By the Implicit Function Theorem,  $\tau(v)$  and  $\beta(v)$  are also well defined and differentiable on  $(v^T, 1)$ . Finally, we notice that  $\alpha(v)$  is uniquely determined by (21).

Proposition 2 will show that the constraint (23) is not binding at this optimal solution. Q.E.D.



*Proof of Lemma 2.* For  $v \in (v_-^h, 1)$ , suppose  $(\tau(v), \alpha(v), \beta(v)) \in \Gamma^T$  and  $(a(v), b(v)) \in \Gamma^P$  are optimal schemes to  $P^T$  and  $P^P$ , respectively.

(i) Let  $\bar{W} = E(W^T | (\tau, \beta), v)$ , as was defined in (22). We have

$$\bar{W}_{\tau\tau}\tau'(v) + \bar{W}_{\tau\beta}\beta'(v) = -[(1 + \lambda)(\tilde{N}'(\tau) - C_y(\tau, L)) + \lambda(C_y(\tau, L) - \beta)], \quad (75)$$

$$\bar{W}_{\beta\tau}\tau'(v) + \bar{W}_{\beta\beta}\beta'(v) = 0. \quad (76)$$

In (71), (72), and (73) it is shown that  $\bar{W}_{\tau\tau} < 0$ ,  $\bar{W}_{\beta\beta} < 0$ , and  $\bar{W}_{\tau\beta} > 0$ . The matrix determinant  $J$  of eqs. (75) and (76) is shown to be positive for all  $v \in (v_-^h, 1)$  [see (74)]. Let  $B$  denote the right-hand side of (75). By Proposition 2,  $B < 0$ . It follows that

$$\tau'(v) = \frac{\begin{vmatrix} B & \bar{W}_{\tau\beta} \\ 0 & \bar{W}_{\beta\beta} \end{vmatrix}}{J} > 0, \quad b'(v) = \frac{\begin{vmatrix} \bar{W}_{\tau\tau} & B \\ \bar{W}_{\beta\tau} & 0 \end{vmatrix}}{J} > 0.$$

(ii) Let  $\hat{W} = E(W^P | b, v)$ , as was defined in (31). We have

$$\begin{aligned} \hat{W}_{bb}b'(v) &= (\tilde{N}'(y(b, H)) - b)y'(b, H) \\ &\quad - (\tilde{N}'(y(b, L)) - b)y'(b, L) - \frac{\lambda}{1 + \lambda}(y(b, H) - y(b, L)). \end{aligned} \quad (77)$$

Let  $D$  denote the right-hand side of (77). From (32) and (33) we can derive that  $D < 0$ . Similar to (i), from the second-order condition  $\hat{W}_{bb} < 0$ ,  $b'(v) = D/\hat{W}_{bb} > 0$ . Q.E.D.

*Proof of Lemma 4.* Write  $\phi = (1 + \lambda)v/(\lambda + v)$ . Let  $y(\cdot, L)$  be the inverse function of  $C_y(\cdot, L)$ . By Assumption 1,  $y'' \leq 0$ . From (24) and by the mean value theorem we have

$$\begin{aligned} \tau &= y(\phi\tilde{N}'(\tau) + (1 - \phi)\beta, L) \\ &\leq y(\beta, L) + y'(\beta, L)\phi(\tilde{N}'(\tau) - \beta). \end{aligned}$$

Thus, from (25):

$$(\tilde{N}'(y(\beta, H)) - \beta)y'(\beta, H) \geq \frac{\lambda}{1 + \lambda}[y(\beta, H) - y(\beta, L) - y'(\beta, L)\phi(\tilde{N}'(\tau) - \beta)]. \quad (78)$$

Multiplying (78) by  $(1-v)$  and adding  $v(\tilde{N}'(\tau)-\beta)y'(\beta, L)$  to both sides yields:

$$v(\tilde{N}'(\tau)-\beta)y'(\beta, L) + (1-v)(\tilde{N}'(y(\beta, H))-\beta)y'(\beta, H) \\ - \frac{\lambda(1-v)}{1+\lambda}(y(\beta, H)-y(\beta, L)) \geq \phi v(\tilde{N}'(\tau)-\beta)y'(\beta, L) > 0.$$

Comparing this inequality with the first-order condition for an optimal price scheme (32) we derive that if  $\beta$  were the price for a price scheme it would not be optimal. By the concavity of  $E(W^P|b, v)$ , therefore,  $\beta(v) < b(v)$ . Q.E.D.

*Proof of Proposition 6.* Let us consider  $E(W^T, x^T | (\tau_1, \beta_1), v_1)$ . Since  $y(\beta_1, H)$ , defined in (20), is continuous in  $\beta_1$  on

$$X = \{(\tau_1, \beta_1) | \tau_1 \geq 0; C_x(\tau_1, H) \leq \beta_1 \leq C_y(\tau_1, L)\},$$

by (51), (53), (55), and (49),  $W_1^T(L)$ ,  $W_1^T(L/H)$ ,  $W_1^T(H)$ , and  $W^T(\tau_1, \beta_1)$  are all continuous in  $(\tau_1, \beta_1)$  on  $X$ . The relationship between  $v_2^T$  and  $(\tau_1, \beta_1)$  is specified by the equilibrium conditions (40), (41), and (43). From the continuity and the strict monotonicity of  $\pi^T(v_2^T)$ , we may define a continuous function  $g(\cdot)$  as the inverse of  $\pi^T(\cdot)$ :  $g(\cdot) = [\pi^T]^{-1}(\cdot)$ . It follows that  $v_2^T(\tau_1, \beta_1)$ , defined as

$$v_2^T(\tau_1, \beta_1) = \begin{cases} v_1, & \Omega^T(\tau_1, \beta_1) \leq \delta \pi^T(v_1), \\ g(\Omega^T(\tau_1, \beta_1)/\delta), & \delta \pi^T(v_1) < \Omega^T(\tau_1, \beta_1) < \delta \pi^T(1^-), \\ 1, & \Omega^T(\tau_1, \beta_1) \geq \delta \pi^T(1^-), \end{cases}$$

is a continuous function from  $X$  onto  $[v_1, 1]$ . On the other hand, from (44) we have  $x^T(v_2^T) = (v_2^T - v_1)/(v_2^T(1 - v_1))$  continuous from  $[v_1, 1]$  onto  $[0, 1]$ . Now it is clear that, being a compound of continuous functions,  $E(W^T, x^T | (\tau_1, \beta_1), v_1)$  is continuous on  $X$ . Note that the target  $\tau_1$  specifies a minimal output. Increasing  $\tau_1$  will eventually lead to a total cost of production (even for agent  $H$ ) higher than the total benefit, making  $E(W^T, x^T | (\tau_1, \beta_1), v_1) < 0$ . Therefore the maximum of the total discounted expected welfare must be attained at some internal points of  $X$ . Eventually this interior solution can be compared with the separating solution,  $(1-v_1)(1+\delta)W^*(H)$ , and the best one leads to a PB equilibrium. Similar arguments apply also to the PIS. Q.E.D.

*Proof of Proposition 7.* Suppose  $v_1 \in V_1 \cap (v^*, 1)$ . According to Proposition 5, there may be three types of equilibria. If  $x^P = 0$ , there is no need for further proof. If  $x^P = 1$ , then  $\Omega(S_1) \geq \delta\pi^P(1^-) > \delta\pi^T(1^-)$ , hence  $x^T = 1$  as well. If  $x^P \in (0, 1)$ , from (43) and the fact that  $v_2^P > v_1 > v^*$ ,

$$\Omega(S_1) = \delta\pi^P(v_2^P) > \delta\pi^T(v_2^P). \quad (79)$$

The posterior belief,  $v_2^T$ , will be determined from

$$v_2^T = \max \{v \in [v^*, 1] \mid \Omega(S_1) \geq \delta\pi^T(v)\}. \quad (80)$$

Eqs. (79) and (80), together with the fact that  $\pi^T(v)$  increases in  $v$ , imply that  $v_2^T > v_2^P$ . It follows from (44) that  $x^T > x^P$ . Q.E.D.

*Proof of Proposition 8.* Let  $EW^T = E(W^T, x \mid (\tau_1, \beta_1), v_1)$ ,  $EW^P = E(W^P, x \mid b_1, v_1)$ . Consider (57) and (58). When  $S_1^h$  is given and fixed,

$$\begin{aligned} \frac{dEW^h}{dx} &= (1 - v_1) \left( W_1^h(H) - W_1^h\left(\frac{L}{H}\right) \right) \\ &\quad + \delta \left[ (v_1 + (1 - v_1)(1 - x)) \frac{dW^h(v_2)}{dv_2} \frac{dv_2}{dx} + (1 - v_1)(W^*(H) - W^h(v_2)) \right]. \end{aligned} \quad (81)$$

When  $\beta_1 \leq \tilde{N}'(y^*(H))$ ,

$$\begin{aligned} &W_1^T(H) - W_1^T(L/H) \\ &= N(y(\beta_1, H)) - C(y(\beta_1, H), H) - \lambda[C(\tau_1, L) + \beta_1(y(\beta_1, H) - \tau_1)] \\ &\quad - [N(\tau_1) - C(\tau_1, H) - \lambda C(\tau_1, L)] \\ &= N(y(\beta_1, H)) - N(\tau_1) - [C(y(\beta_1, H), H) - C(\tau_1, H)] \\ &\quad - \lambda\beta_1(y(\beta_1, H) - \tau_1) \\ &> (1 + \lambda)[(\tilde{N}(y(\beta_1, H)) - \tilde{N}(\tau)) - \beta_1(y(\beta_1, H) - \tau_1)] \\ &> (1 + \lambda)(\tilde{N}'(y^*(H)) - \beta_1)(y(\beta_1, H) - \tau_1) \end{aligned}$$

$$\geq 0.$$

Similarly,  $W_1^p(H) - W_1^p(L/H) > 0$  for  $b_1 \leq \tilde{N}'(y^*(H))$ . Now write  $W^h(v_2) = v_2 W_2^h(L) + (1 - v_2) W_2^h(H)$ , with  $W_2^h(\theta)$ ,  $\theta = L, H$ , denoting the welfare with the second-period optimal scheme,  $S_2^h(v_2)$ , when the agent's type is  $\theta$ . By the envelope theorem,

$$\frac{dW^h(v_2)}{dv_2} = W_2^h(L) - W_2^h(H). \quad (82)$$

We also have

$$\frac{dv_2}{dx} = \frac{v_1(1 - v_1)}{(v_1 + (1 - v_1)(1 - x))^2}. \quad (83)$$

Substituting (82) and (83) into (81) gives

$$\frac{dEW^h}{dx} = (1 - v_1)[(W_1^h(H) - W_1^h(L/H)) + \delta(W^*(H) - W_2^h(H))] > 0,$$

since  $W^*(H) \geq W_2^h(H)$ . Q.E.D.

*Proof of Lemma 8.* Substitute (59) into (50)–(55). By comparing terms we have  $W_1^T(L) = W_1^p(L)$ ,  $W_1^T(L/H) = W_1^p(L/H)$ ,  $W_1^T(H) \geq W_1^p(H)$ , and  $W^T(v_2) \geq W^p(v_2)$ , where the last inequality holds strictly for  $v_2 < 1$ , i.e. for  $x < 1$ . It follows immediately that

$$\begin{aligned} & E(W^T, x | (\tau_1, \beta_1), v_1) - E(W^p, x | b_1, v_1) \\ &= (1 - v_1)x(W_1^T(H) - W_1^p(H)) \\ & \quad + \delta[v_1 + (1 - v_1)(1 - x)](W^T(v_2) - W^p(v_2)) \\ & \geq 0, \end{aligned}$$

with strict inequality for  $x < 1$ . Q.E.D.

*Proof of Proposition 9.* First consider the case where  $(y^*, 1) \subseteq V_1$ .

(a) Suppose that the equilibrium with  $(a_1(v_1), b_1(v_1))$  is pooling. This means that the agent  $H$ 's first-period strategy is  $x^p = 0$ . Consider a target-



incentive scheme  $(\hat{\tau}_1, \hat{\alpha}_1, \hat{\beta}_1)$  defined in (59) with respect to  $(a_1, b_1)$ . Since  $\hat{\beta}_1 \leq \hat{N}'(y^*(H))$ , from Proposition 8 and Lemma 8:

$$E(W^T, x^T | (\hat{\tau}_1, \hat{\beta}_1), v_1) \geq E(W^T, 0 | (\hat{\tau}_1, \hat{\beta}_1), v_1) > E(W^P, 0 | b_1, v_1),$$

hence (60).

(b) Suppose that the equilibrium with  $(a_1(v_1), b_1(v_1))$  is separating. Let  $(\hat{\tau}_1, \hat{\alpha}_1, \hat{\beta}_1)$  be such that

$$\hat{\tau}_1 = y(b_1, L),$$

$$\hat{\alpha}_1 = a_1 + b_1 y(b_1, L), \quad (84)$$

$$\hat{\beta}_1 = b_1.$$

By Proposition 5 and Lemma 5:

$$\Omega^T(\hat{\tau}_1, \hat{\beta}_1) = \Omega^P(b_1) \geq \delta \pi^P(1^-) > \delta \pi^T(1^-).$$

Therefore  $(\hat{\tau}_1, \hat{\alpha}_1, \hat{\beta}_1)$  also achieves a separating equilibrium. It is straightforward to verify that

$$E(W_1^T, 1 | (\hat{\tau}_1, \hat{\beta}_1), v_1) = E(W^P, 1 | b_1, v_1). \quad (85)$$

In a separating equilibrium the total discounted expected welfare reduces to

$$\begin{aligned} E(W_1^T, 1 | (\hat{\tau}_1, \hat{\beta}_1), v_1) &= v_1 W_1^T(L) + (1 - v_1) W_1^T(H) \\ &\quad + \delta [v_1 W^*(L) + (1 - v_1) W^*(H)]. \end{aligned}$$

The first part,  $v_1 W_1^T(L) + (1 - v_1) W_1^T(H)$ , is identical to the static expected welfare in (15). By Proposition 4,  $(\hat{\tau}_1, \hat{\alpha}_1, \hat{\beta}_1)$  is not statically optimal. Noting that the remaining part,  $\delta [v_1 W^*(L) + (1 - v_1) W^*(H)]$ , is a constant, by the continuity of  $W^T$  we can solve the program

$$\max_{(\tau_1, \alpha_1, \beta_1)} v_1 W_1^T(L) + (1 - v_1) W_1^T(H)$$

$$\text{s.t. } \Omega^T(\tau_1, \beta_1) \geq \delta \pi^T(1^-)$$

to find another first-period target-incentive scheme  $(\tau_1, \alpha_1, \beta_1)$  such that  $E(W^T, 1 | (\tau_1, \beta_1), v_1) > E(W^T, 1 | (\hat{\tau}_1, \hat{\beta}_1), v_1)$ . Therefore (60) must hold.

(c) Suppose  $(a_1(v_1), b_1(v_1))$  induces a semi-separating equilibrium. Let  $(\hat{\tau}_1, \hat{\alpha}_1, \hat{\beta}_1)$  be defined as in (84). By Proposition 7 and Proposition 5:

$$\Omega^T(\hat{\tau}_1, \hat{\beta}_1) = \Omega^P(b_1) = \delta\pi^P(v_2^P) > \delta\pi^T(v_2^P), \quad (86)$$

since  $v_2^P > v_1 \geq \hat{v}$ .

(i) First consider  $\hat{\beta}_1 \leq \tilde{N}'(y^*(H))$ . The continuation equilibrium requires that the principal's posterior belief,  $v_2^T$ , in the TIS be determined from

$$v_2^T = \max \{v \in [v_2^P, 1] \mid \Omega^T(\hat{\tau}_1, \hat{\beta}_1) \geq \delta\pi^T(v)\}.$$

Eq. (86) and the increasing property of  $\pi^*$  indicate that  $v_2^T > v_2^P$ , hence  $x^T > x^P$ . Expression (60) follows readily from Proposition 8.

(ii) Next consider  $\hat{\beta}_1 > \tilde{N}'(y^*(H))$ . Define a new piece-bonus  $\tilde{\beta}$  as

$$\tilde{\beta} = \min \{\beta \mid \beta \geq \tilde{N}'(y^*(H)); \Omega^T(\hat{\tau}_1, \beta) \geq \delta\pi^T(v_2^P)\}.$$

Differentiating (49) yields  $\Omega_{\beta}^T(\hat{\tau}_1, \hat{\beta}_1) = y(\hat{\beta}_1, H) - \hat{\tau}_1 > 0$ , thus  $\tilde{\beta} < \hat{\beta}_1$ . The posterior belief with the first-period target-incentive scheme  $(\hat{\tau}_1, \hat{\alpha}_1, \tilde{\beta})$  will be

$$v_2^T = \max \{v \in [v_2^P, 1] \mid \Omega(\hat{\tau}_1, \tilde{\beta}) \geq \delta\pi^T(v)\}. \quad (87)$$

If  $\tilde{\beta} > \tilde{N}'(y^*(H))$ , then  $\Omega^T(\hat{\tau}_1, \tilde{\beta}) = \delta\pi^T(v_2^P)$ , and by (87),  $v_2^T = v_2^P$ . Therefore the corresponding strategy is  $x^T = x^P$ . Since  $\tilde{\beta} < \hat{\beta}_1$ , from Lemma 8 we know that (60) holds. If  $\tilde{\beta} = \tilde{N}'(y^*(H))$ , then  $\Omega^T(\hat{\tau}_1, \tilde{\beta}) > \delta\pi^T(v_2^P)$  and  $x^T > x^P$ . From Proposition 8 and Lemma 8 it follows that

$$E(W^T, x^T \mid (\hat{\tau}_1, \tilde{\beta}), v_1) > E(W^T, x^P \mid (\hat{\tau}_1, \tilde{\beta}), v_1) > E(W^T, x^P \mid b_1, v_1).$$

(d) By the definition of  $V_1^H$  and from what we have shown, it follows that  $V_1^P \cap (y^*, 1) \subseteq V_1^T \cap (y^*, 1)$  and the set inclusion is strict if  $V_1^P \subset (y^*, 1)$ . Thus, if  $v_1 \notin V_1^T$ , then  $v_1 \notin V_1^P$ , and in both systems the principal achieves the same welfare level  $(1 - v_1)(1 + \delta)W^*(H)$ . Finally, if  $v_1 \in V_1^T$  but  $v_1 \notin V_1^P$ , then by the very definition of  $V_1^H$  the strict inequality in (61) still holds.

(e) The case with  $v_1 = 0$  or  $v_1 = 1$  is trivial. Q.E.D.

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